OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** Roger of mass 70 kg and Sheuli of mass 50 kg are skating on a horizontal plane containing the standard unit vectors **i** and **j**. The resistances to the motion of the skaters are negligible. The two skaters are locked in a close embrace and accelerate from rest until they reach a velocity of $2\mathbf{i}\ \mathrm{m\,s^{-1}}$, as shown in Fig. 1.1.

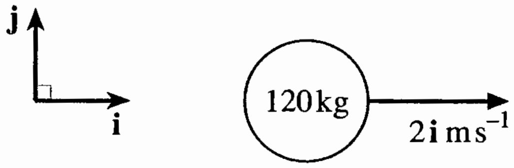

Fig. 1.1

(i) What impulse has acted on them? [1]

During a dance routine, the skaters separate on three occasions from their close embrace when travelling at a constant velocity of $2\mathbf{i}\ \mathrm{m\,s^{-1}}$.

(ii) Calculate the velocity of Sheuli after the separation in the following cases.

(*A*) Roger has velocity $\mathbf{i}\ \mathrm{m\,s^{-1}}$ after the separation.

(*B*) Roger and Sheuli have equal speeds in opposite senses after the separation, with Roger moving in the **i** direction.

(*C*) Roger has velocity $4(\mathbf{i}+\mathbf{j})\ \mathrm{m\,s^{-1}}$ after the separation. [6]

(b) Two discs with masses 2 kg and 3 kg collide directly in a horizontal plane. Their velocities just before the collision are shown in Fig. 1.2. The coefficient of restitution in the collision is 0.5.

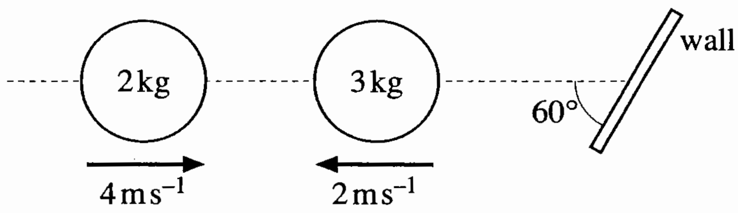

Fig. 1.2

(i) Calculate the velocity of each disc after the collision. [6]

The disc of mass 3 kg moves freely after the collision and makes a perfectly elastic collision with a smooth wall inclined at 60° to its direction of motion, as shown in Fig. 1.2.

(ii) State with reasons the speed of the disc and the angle between its direction of motion and the wall after the collision. [4]

2 A car of mass 850 kg is travelling along a road that is straight but not level.

On one section of the road the car travels at constant speed and gains a vertical height of 60 m in 20 seconds. Non-gravitational resistances to its motion (e.g. air resistance) are negligible.

(i) Show that the average power produced by the car is about 25 kW. [2]

On a *horizontal* section of the road, the car develops a constant power of exactly 25 kW and there is a constant resistance of 800 N to its motion.

(ii) Calculate the maximum possible steady speed of the car. [3]

(iii) Find the driving force and acceleration of the car when its speed is $10\,\text{m}\,\text{s}^{-1}$. [3]

When travelling along the horizontal section of road, the car accelerates from $15\,\text{m}\,\text{s}^{-1}$ to $20\,\text{m}\,\text{s}^{-1}$ in 6.90 seconds with the same constant power and constant resistance.

(iv) By considering work and energy, find how far the car travels while it is accelerating. [6]

When the car is travelling at $20\,\text{m}\,\text{s}^{-1}$ up a constant slope inclined at arcsin (0.05) to the horizontal, the driving force is removed. Subsequently, the resistance to the motion of the car remains constant at 800 N.

(v) What is the speed of the car when it has travelled a further 105 m up the slope? [5]

3 Fig. 3.1 shows an object made up as follows. ABCD is a uniform lamina of mass 16 kg. BE, EF, FG, HI, IJ and JD are each uniform rods of mass 2 kg. ABCD, BEFG and HIJD are squares lying in the same plane. The dimensions in metres are shown in the figure.

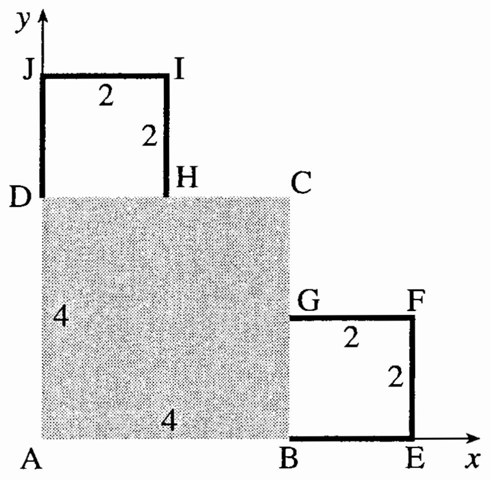

Fig. 3.1

(i) Find the coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.1. [5]

The rods are now re-positioned so that BEFG and HIJD are perpendicular to the lamina, as shown in Fig. 3.2.

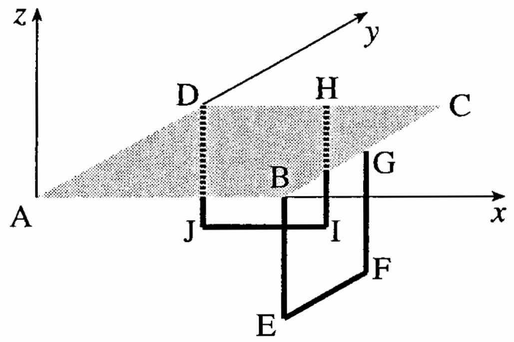

Fig. 3.2

(ii) Find the x-, y- and z-coordinates of the centre of mass of the object, referred to the axes shown in Fig. 3.2. Calculate the distance of the centre of mass from A. [8]

The object is now freely suspended from A and hangs in equilibrium with AC at $\alpha°$ to the vertical.

(iii) Calculate α. [4]

4 **(a)** A framework is made from light rods AB, BC and CA. They are freely hinged to each other at A, B and C and to a vertical wall at A. The hinge at B rests on a smooth, horizontal support. The rod AC is horizontal. A vertical load of L N acts at C. This information is shown in Fig. 4.1 together with the dimensions of the framework and the external forces U N, V N and R N acting on the framework.

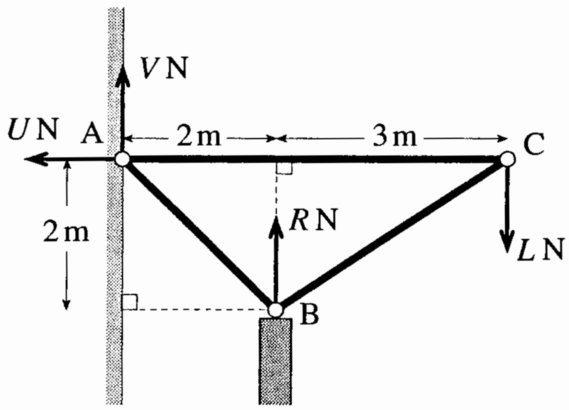

Fig. 4.1

(i) Show that $R = 2.5L$, $U = 0$ and $V = -1.5L$. [4]

(ii) Calculate the internal forces in the rods AB, AC and BC in terms of L, stating whether each of these rods is in tension or thrust (compression). [8]

(b) Fig. 4.2 shows a plank of weight W resting at the points A and B on two fixed supports. The plank is at an angle θ to the horizontal. Its centre of mass, G, is such that AG is 2 m and GB is 1 m.

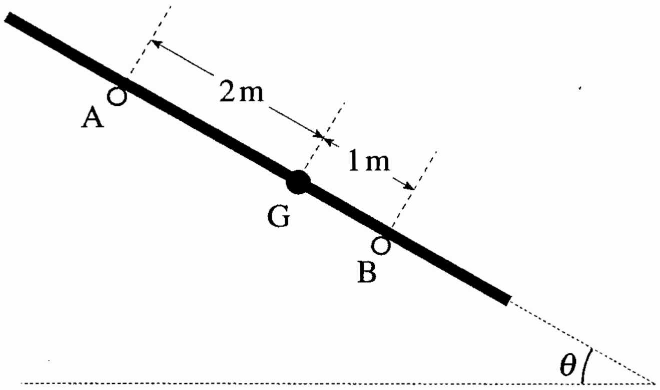

Fig. 4.2

The contact between the plank and the support at A is rough, but that at B is smooth. The plank is on the point of slipping.

(i) Draw a diagram showing all the forces acting on the plank. [1]

(ii) By taking moments about a suitable point, find an expression in terms of W and θ for the normal reaction at A of the support on the plank. [3]

(iii) Find an expression in terms of θ for the coefficient of friction between the plank and the rough support. [3]

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Friday **27 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
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This question paper consists of 6 printed pages and 2 blank pages.

1 When a stationary firework P of mass 0.4 kg is set off, the explosion gives it an instantaneous impulse of 16 N s vertically upwards.

(i) Calculate the speed of projection of P. [2]

While travelling vertically upwards at $32\,\text{m s}^{-1}$, P collides directly with another firework Q, of mass 0.6 kg, that is moving directly downwards with speed $u\,\text{m s}^{-1}$, as shown in Fig. 1. The coefficient of restitution in the collision is 0.1 and Q has a speed of $4\,\text{m s}^{-1}$ vertically *upwards* immediately after the collision.

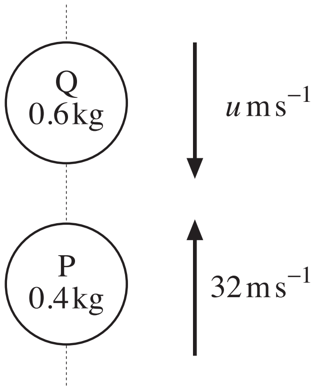


Fig. 1

(ii) Show that $u = 18$ and calculate the speed and direction of motion of P immediately after the collision. [7]

Another firework of mass 0.5 kg has a velocity of $(-3.6\mathbf{i} + 5.2\mathbf{j})\,\text{m s}^{-1}$, where **i** and **j** are horizontal and vertical unit vectors, respectively. This firework explodes into two parts, C and D. Part C has mass 0.2 kg and velocity $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ immediately after the explosion.

(iii) Calculate the velocity of D immediately after the explosion in the form $a\mathbf{i} + b\mathbf{j}$. Show that C and D move off at 90° to one another. [8]

2 A uniform beam, AB, is 6 m long and has a weight of 240 N.

Initially, the beam is in equilibrium on two supports at C and D, as shown in Fig. 2.1. The beam is horizontal.

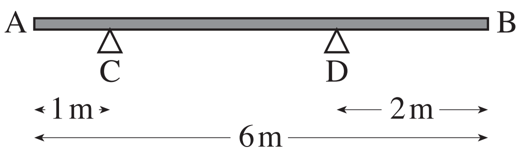


Fig. 2.1

(i) Calculate the forces acting on the beam from the supports at C and D. [4]

A workman tries to move the beam by applying a force T N at A at 40° to the beam, as shown in Fig. 2.2. The beam remains in horizontal equilibrium but the reaction of support C on the beam is zero.

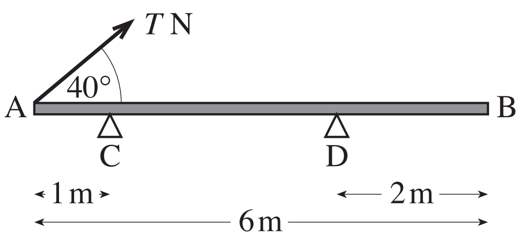


Fig. 2.2

(ii) (*A*) Calculate the value of T. [4]

(*B*) Explain why the support at D cannot be smooth. [1]

The beam is now supported by a light rope attached to the beam at A, with B on rough, horizontal ground. The rope is at 90° to the beam and the beam is at 30° to the horizontal, as shown in Fig. 2.3. The tension in the rope is P N. The beam is in equilibrium on the point of sliding.

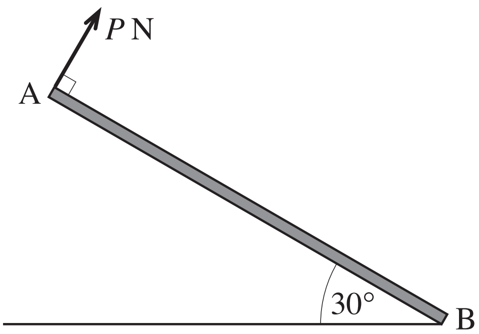


Fig. 2.3

(iii) (*A*) Show that $P = 60\sqrt{3}$ and hence, or otherwise, find the frictional force between the beam and the ground. [5]

(*B*) Calculate the coefficient of friction between the beam and the ground. [5]

[Turn over

3 **(a)** A uniform lamina made from rectangular parts is shown in Fig. 3.1. All the dimensions are centimetres. All coordinates are referred to the axes shown in Fig. 3.1.

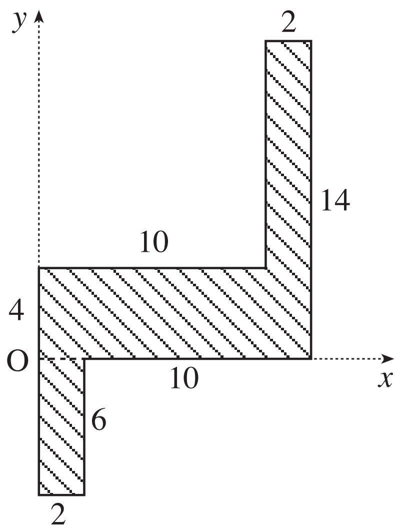


Fig. 3.1

(i) Show that the x-coordinate of the centre of mass of the lamina is 6.5 and find the y-coordinate. [5]

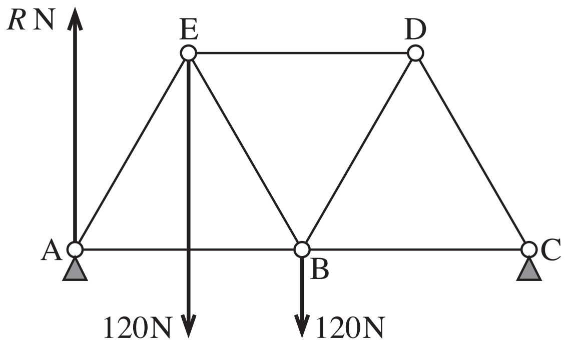
A square of side 2 cm is to be cut from the lamina. The sides of the square are to be parallel to the coordinate axes and the centre of the square is to be chosen so that the x-coordinate of the centre of mass of the new shape is 6.4.

(ii) Calculate the x-coordinate of the centre of the square to be removed. [3]

The y-coordinate of the centre of the square to be removed is now chosen so that the y-coordinate of the centre of mass of the final shape is as large as possible.

(iii) Calculate the y-coordinate of the centre of mass of the lamina with the square removed, giving your answer correct to three significant figures. [3]

(b) Fig. 3.2 shows a framework made from light rods of length 2 m freely pin-jointed at A, B, C, D and E. The framework is in a vertical plane and is supported at A and C. There are loads of 120 N at B and at E. The force on the framework due to the support at A is R N.



each rod is 2 m long

Fig. 3.2

(i) Show that $R = 150$. [2]

(ii) Draw a diagram showing all the forces acting at the points A, B, D and E, including the forces internal to the rods.

Calculate the internal forces in rods AE and EB, and determine whether each is a tension or a thrust. [You may leave your answers in surd form.] [6]

(iii) Without any further calculation of the forces in the rods, explain briefly how you can tell that rod ED is in thrust. [1]

[Question 4 is printed overleaf.]

4 A block of mass 20 kg is pulled by a light, horizontal string over a rough, horizontal plane. During 6 seconds, the work done against resistances is 510 J and the speed of the block increases from $5\,\mathrm{m\,s^{-1}}$ to $8\,\mathrm{m\,s^{-1}}$.

(i) Calculate the power of the pulling force. [4]

The block is now put on a rough plane that is at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The frictional resistance to sliding is $11g$ N. A light string parallel to the plane is connected to the block. The string passes over a smooth pulley and is connected to a freely hanging sphere of mass m kg, as shown in Fig. 4.

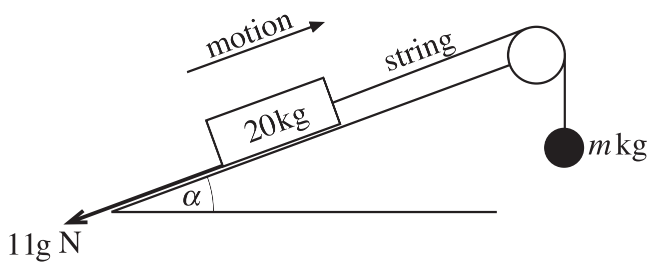


Fig. 4

In parts **(ii)** and **(iii)**, the sphere is pulled downwards and then released when travelling at a speed of $4\,\mathrm{m\,s^{-1}}$ vertically downwards. The block never reaches the pulley.

(ii) Suppose that $m = 5$ and that after the sphere is released the block moves x m up the plane before coming to rest.

(A) Find an expression in terms of x for the change in gravitational potential energy of the system, stating whether this is a gain or a loss. [4]

(B) Find an expression in terms of x for the work done against friction. [1]

(C) Making use of your answers to parts (A) and (B), find the value of x. [3]

(iii) Suppose instead that $m = 15$. Calculate the speed of the sphere when it has fallen a distance 0.5 m from its point of release. [4]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS 4762

Mechanics 2

Monday **19 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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1 **(a)** Two small spheres, P of mass 2 kg and Q of mass 6 kg, are moving in the same straight line along a smooth, horizontal plane with the velocities shown in Fig. 1.1.

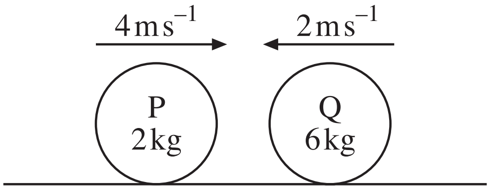

Fig. 1.1

Consider the direct collision of P and Q in the following two cases.

(i) The spheres coalesce on collision.

(*A*) Calculate the common velocity of the spheres after the collision. [3]

(*B*) Calculate the energy lost in the collision. [2]

(ii) The spheres rebound with a coefficient of restitution of $\frac{2}{3}$ in the collision.

(*A*) Calculate the velocities of P and Q after the collision. [6]

(*B*) Calculate the impulse on P in the collision. [2]

(b) A small ball bounces off a smooth, horizontal plane. The ball hits the plane with a speed of $26\,\mathrm{m\,s^{-1}}$ at an angle of $\arcsin \frac{12}{13}$ to it. The ball rebounds at an angle of $\arcsin \frac{3}{5}$ to the plane, as shown in Fig. 1.2.

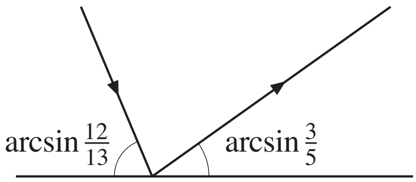

Fig. 1.2

Calculate the speed with which the ball rebounds from the plane.

Calculate also the coefficient of restitution in the impact. [6]

[Turn over

2 Two heavy rods AB and BC are freely jointed together at B and to a wall at A. AB has weight 90 N and centre of mass at P; BC has weight 75 N and centre of mass at Q. The lengths of the rods and the positions of P and Q are shown in Fig. 2.1, with the lengths in metres.

Initially, AB and BC are horizontal. There is a support at R, as shown in Fig. 2.1. The system is held in equilibrium by a vertical force acting at C.

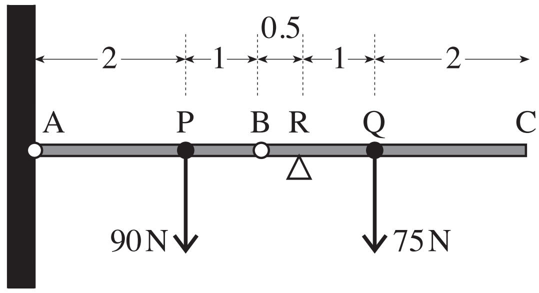

Fig. 2.1

(i) Draw diagrams showing all the forces acting on rod AB and on rod BC.

Calculate the force exerted on AB by the hinge at B and hence the force required at C. [6]

The rods are now set up as shown in Fig. 2.2. AB and BC are each inclined at 60° to the vertical and C rests on a rough horizontal table. Fig. 2.3 shows all the forces acting on AB, including the forces X N and Y N due to the hinge at A and the forces U N and V N in the hinge at B. The rods are in equilibrium.

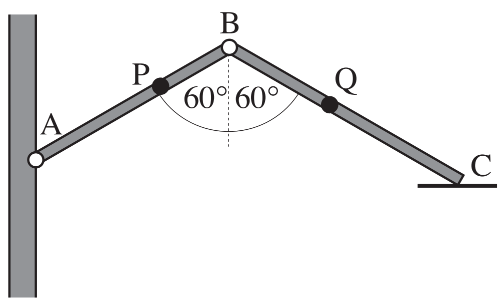

Fig. 2.2

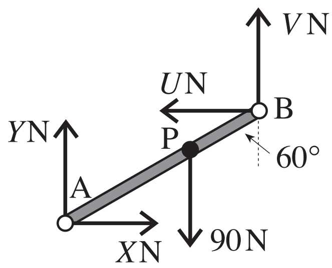

Fig. 2.3

(ii) By considering the equilibrium of rod AB, show that $60\sqrt{3} = U + V\sqrt{3}$. [3]

(iii) Draw a diagram showing all the forces acting on rod BC. [1]

(iv) Find a further equation connecting U and V and hence find their values. Find also the frictional force at C. [8]

3 **(a)** A car of mass 900 kg is travelling at a steady speed of 16 m s^{-1} up a hill inclined at $\arcsin 0.1$ to the horizontal. The power required to do this is 20 kW.

Calculate the resistance to the motion of the car. [4]

(b) A small box of mass 11 kg is placed on a uniform rough slope inclined at $\arccos \frac{12}{13}$ to the horizontal. The coefficient of friction between the box and the slope is μ.

(i) Show that if the box stays at rest then $\mu \geqslant \frac{5}{12}$. [3]

For the remainder of this question, the box moves on a part of the slope where $\mu = 0.2$.

The box is projected up the slope from a point P with an initial speed of $v \text{ m s}^{-1}$. It travels a distance of 1.5 m along the slope before coming instantaneously to rest. During this motion, the work done against air resistance is 6 joules per metre.

(ii) Calculate the value of v. [5]

As the box slides back down the slope, it passes through its point of projection P and later reaches its initial speed at a point Q. During this motion, once again the work done against air resistance is 6 joules per metre.

(iii) Calculate the distance PQ. [6]

[Question 4 is printed overleaf.]

4 Fig. 4.1 shows four uniform rods, OA, AB, BE and CD, rigidly fixed together to form a frame. The rods have weights proportional to their lengths and these lengths, in centimetres, are shown in Fig. 4.1.

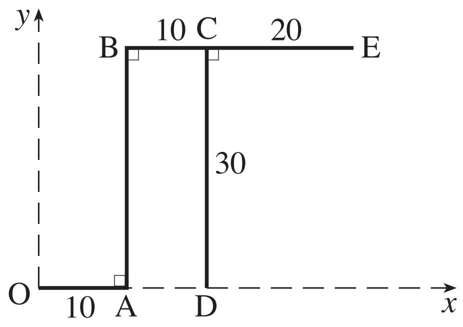

Fig. 4.1

(i) Calculate the coordinates of the centre of mass of the frame, referred to the axes shown in Fig. 4.1. [5]

The bracket shown in Fig. 4.2 is made of uniform sheet metal with cross-section the frame shown in Fig. 4.1. The bracket is 40 cm wide and its weight is 60 N. It stands on a horizontal plane containing Ox and Oz.

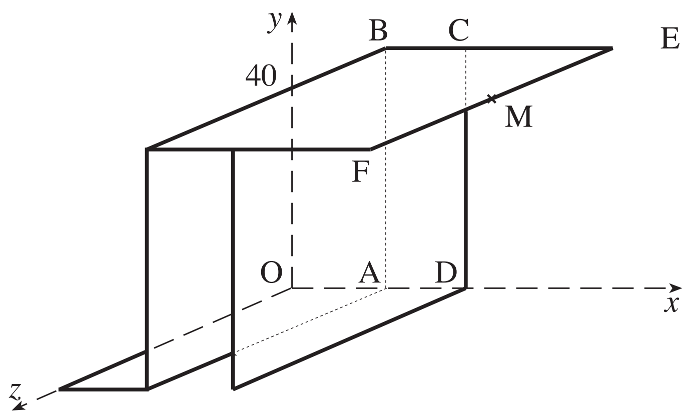

Fig. 4.2

(ii) Write down the coordinates of the centre of mass of the bracket, referred to the axes shown in Fig. 4.2. [2]

A force P N acts vertically downwards at the point M, shown in Fig. 4.2. M is the mid-point of EF. The bracket is on the point of tipping.

(iii) Calculate the value of P. [4]

In another situation, a horizontal force Q N acts through M parallel to EB and in the direction from E to B. The value of Q is increased from zero with the bracket in equilibrium at all times.

(iv) Draw a diagram showing the forces acting on the bracket when it is on the point of tipping. [1]

(v) If the limiting frictional force between the bracket and the plane is 30 N, does the bracket slide or tip first as Q is increased? [5]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4762/01

MATHEMATICS (MEI)

Mechanics 2

WEDNESDAY 10 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

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- Answer **all** the questions.
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INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
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This document consists of **7** printed pages and **1** blank page.

1 A sledge and a child sitting on it have a combined mass of 29.5 kg. The sledge slides on horizontal ice with negligible resistance to its movement.

(i) While at rest, the sledge is hit directly from behind by a ball of mass 0.5 kg travelling horizontally at 10 m s^{-1}. The coefficient of restitution in the collision is 0.8. After the impact the speeds of the sledge and the ball are $V_1 \text{ m s}^{-1}$ and $V_2 \text{ m s}^{-1}$ respectively.

Calculate V_1 and V_2 and state the direction in which the ball is travelling after the impact. [7]

(ii) While at rest, the sledge is hit directly from behind by a snowball of mass 0.5 kg travelling horizontally at 10 m s^{-1}. The snowball sticks to the sledge.

(*A*) Calculate the velocity with which the combined sledge and snowball start to move. [3]

(*B*) The child scoops up the 0.5 kg of snow and drops it over the back of the sledge. What happens to the velocity of the sledge? Give a reason for your answer. [2]

(iii) In another situation, the sledge is travelling over the ice at 2 m s^{-1} with 10.5 kg of snow on it (giving a total mass of 40 kg). The child throws a snowball of mass 0.5 kg from the sledge, parallel to the ground and in the positive direction of the motion of the sledge. Immediately after the snowball is thrown, the sledge has a speed of $V \text{ m s}^{-1}$ and the snowball and sledge are separating at a speed of 10 m s^{-1}.

Draw a diagram showing the velocities of the sledge and snowball before and after the snowball is thrown.

Calculate V. [5]

2

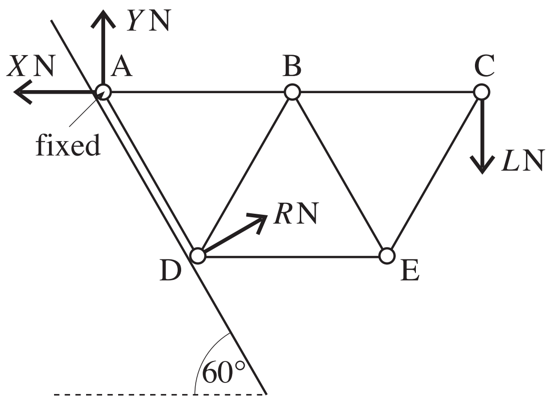

Fig. 2

Fig. 2 shows a framework in a vertical plane made from the equal, light, rigid rods AB, BC, AD, BD, BE, CE and DE. [The triangles ABD, BDE and BCE are all equilateral.]

The rods AB, BC and DE are horizontal.

The rods are freely pin-jointed to each other at A, B, C, D and E.

The pin-joint at A is also fixed to an inclined plane. The plane is smooth and parallel to the rod AD. The pin-joint at D rests on this plane.

The following external forces act on the framework: a vertical load of L N at C; the normal reaction force R N of the plane on the framework at D; the horizontal and vertical forces X N and Y N, respectively, acting at A.

(i) Write down equations for the horizontal and vertical equilibrium of the framework. [3]

(ii) By considering moments, find the relationship between R and L. Hence show that $X = \sqrt{3}L$ and $Y = 0$. [4]

(iii) Draw a diagram showing all the forces acting on the pin-joints, including the forces internal to the rods. [2]

(iv) Show that the internal force in the rod AD is zero. [2]

(v) Find the forces internal to AB, CE and BC in terms of L and state whether each is a tension or a thrust (compression). [You may leave your answers in surd form.] [7]

(vi) Without calculating their values in terms of L, show that the forces internal to the rods BD and BE have equal magnitude but one is a tension and the other a thrust. [2]

[Turn over

3 A box is to be assembled in the shape of the cuboid shown in Fig. 3.1. The lengths are in centimetres. All the faces are made of the same uniform, rigid and thin material. All coordinates refer to the axes shown in this figure.

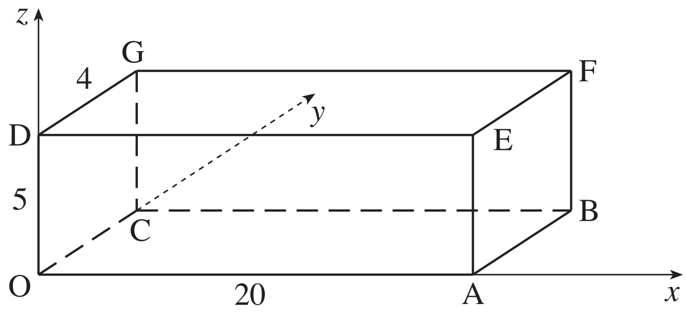

Fig. 3.1

(i) The four vertical faces OAED, ABFE, FGCB and CODG are assembled first to make an open box without a base or a top. Write down the coordinates of the centre of mass of this open box. [1]

The base OABC is added to the vertical faces.

(ii) Write down the x- and y-coordinates of the centre of mass of the box now. Show that the z-coordinate is now 1.875. [5]

The top face FGDE is now added. This is a lid hinged to the rest of the box along the line FG. The lid is open so that it hangs in a vertical plane touching the face FGCB.

(iii) Show that the coordinates of the centre of mass of the box in this situation are $(10, 2.4, 2.1)$. [6]

[This question is continued on the facing page.]

The box, with the lid still touching face FGCB, is now put on a sloping plane with the edge OA horizontal and the base inclined at 30° to the horizontal, as shown in Fig. 3.2.

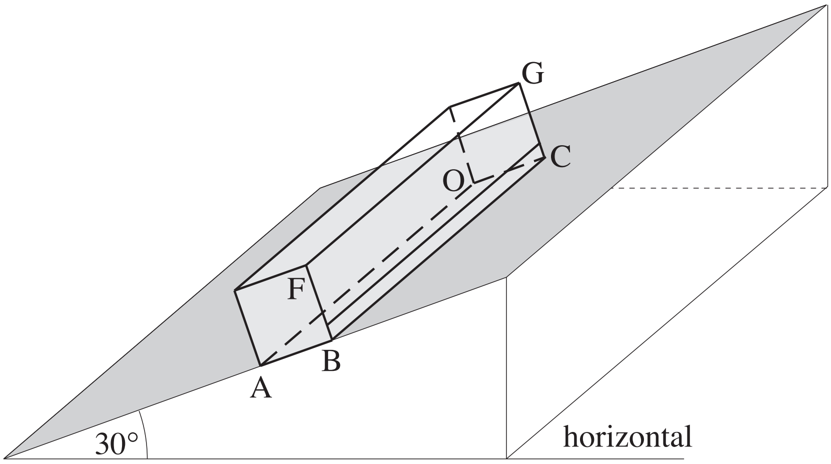

Fig. 3.2

The weight of the box is 40 N. A force P N acts parallel to the plane and is applied to the mid-point of FG at 90° to FG. This force tends to push the box down the plane. The box does not slip and is on the point of toppling about the edge AO.

(iv) Show that the clockwise moment of the weight of the box about the edge AO is about 0.411 N m. [4]

(v) Calculate the value of P. [2]

[Question 4 is printed overleaf.]

4 Some tiles on a roof are being replaced. Each tile has a mass of 2 kg and the coefficient of friction between it and the existing roof is 0.75. The roof is at 30° to the horizontal and the bottom of the roof is 6 m above horizontal ground, as shown in Fig. 4.

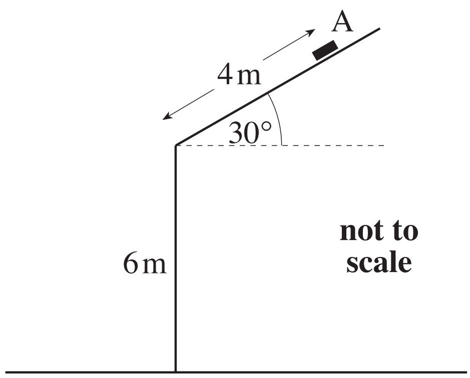

Fig. 4

(i) Calculate the limiting frictional force between a tile and the roof.

A tile is placed on the roof. Does it slide? (Your answer should be supported by a calculation.) [5]

(ii) The tiles are raised 6 m from the ground, the only work done being against gravity. They are then slid 4 m up the roof and placed at the point A shown in Fig. 4.

(*A*) Show that each tile gains 156.8 J of gravitational potential energy. [3]

(*B*) Calculate the work done against friction per tile. [2]

(*C*) What average power is required to raise 10 tiles per minute from the ground to A? [2]

(iii) A tile is kicked from A directly down the roof. When the tile is at B, x m from the edge of the roof, its speed is $4\,\text{m s}^{-1}$. It subsequently hits the ground travelling at $9\,\text{m s}^{-1}$. In the motion of the tile from B to the ground, the work done against sliding and other resistances is 90 J.

Use an energy method to find x. [5]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4762/01**

MATHEMATICS (MEI)

Mechanics 2

WEDNESDAY 20 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

  [Turn over

1 **(a)** Disc A of mass 6 kg is on a smooth horizontal plane. The disc is at rest and then a constant force of magnitude 9 N acts on it for 2 seconds.

(i) Find the magnitude of the impulse of the force on the disc. Hence, or otherwise, find the speed of the disc after the two seconds. [2]

Without losing speed, disc A now collides directly with disc B of mass 2 kg which is also on the plane. Just before the collision, disc B is travelling at 1 m s^{-1} in the opposite direction to the motion of A, as shown in Fig. 1.1. On impact the two discs stick together to form the combined object AB.

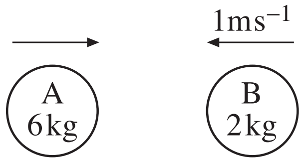

Fig. 1.1

(ii) Show that AB moves off with a speed of 2 m s^{-1} in the original direction of motion of disc A. [3]

(iii) Find the impulse that acts on disc B in the collision. [2]

The combined object AB now collides directly with disc C of mass 10 kg, which is moving on the plane at 1.8 m s^{-1} in the same direction as AB. After this collision the speed of AB is $v \text{ m s}^{-1}$ in the same direction as its speed before the impact, and disc C moves off with speed 1.9 m s^{-1}.

(iv) (*A*) Draw a diagram indicating the velocities before and after the collision. [1]

(*B*) Calculate the value of v. [3]

(*C*) Calculate the coefficient of restitution in the collision. [3]

(b) A small ball is thrown horizontally with a speed of 8 m s^{-1}. The point of projection is 10 m above a smooth horizontal plane, as shown in Fig. 1.2.

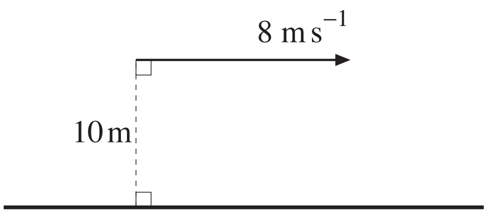

Fig. 1.2

The coefficient of restitution in the impact between the ball and the plane is $\frac{4}{7}$.

Calculate the vertical component of the velocity of the ball immediately after its first impact with the plane and also the angle at which the ball rebounds from the plane. [5]

2 The position of the centre of mass, G, of a uniform wire bent into the shape of an arc of a circle of radius r and centre C is shown in Fig. 2.1.

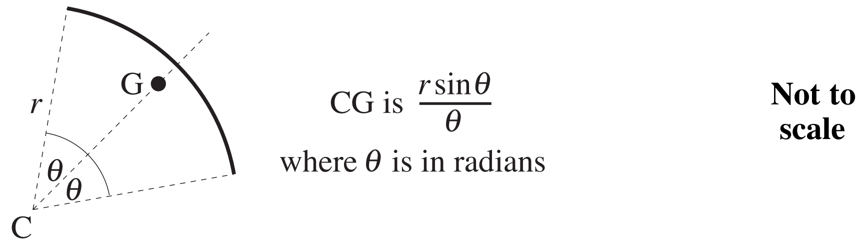

Fig. 2.1

(i) Use this information to show that the centre of mass, G, of the uniform wire bent into the shape of a semi-circular arc of radius 8 shown in Fig. 2.2 has coordinates $\left(-\frac{16}{\pi}, 8\right)$. [3]

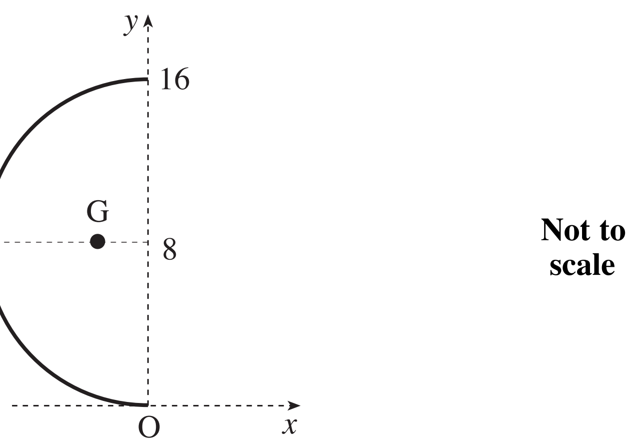

Fig. 2.2

A walking-stick is modelled as a uniform rigid wire. The walking-stick and coordinate axes are shown in Fig. 2.3. The section from O to A is a semi-circular arc and the section OB lies along the x-axis. The lengths are in centimetres.

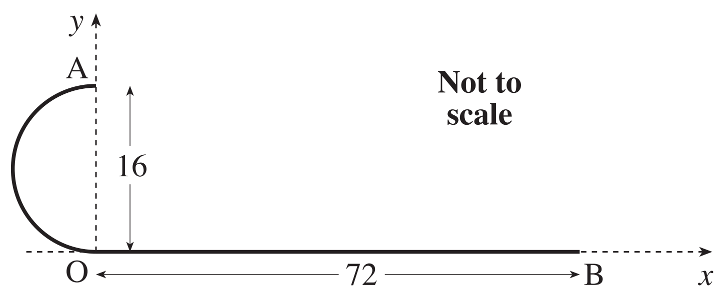

Fig. 2.3

(ii) Show that the coordinates of the centre of mass of the walking-stick are $(25.37, 2.07)$, correct to two decimal places. [6]

The walking-stick is now hung from a shelf as shown in Fig. 2.4. The only contact between the walking-stick and the shelf is at A.

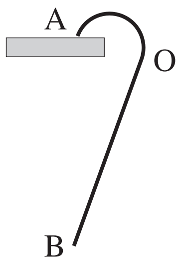

Fig. 2.4

(iii) When the walking-stick is in equilibrium, OB is at an angle α to the vertical.

Draw a diagram showing the position of the centre of mass of the walking-stick in relation to A.

Calculate α. [5]

(iv) The walking-stick is now held in equilibrium, with OB vertical and A still resting on the shelf, by means of a vertical force, F N, at B. The weight of the walking-stick is 12 N. Calculate F. [3]

3 A uniform plank is 2.8 m long and has weight 200 N. The centre of mass is G.

(i) Fig. 3.1 shows the plank horizontal and in equilibrium, resting on supports at A and B.

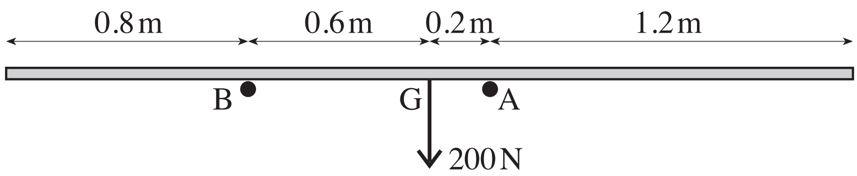

Fig. 3.1

Calculate the reactions of the supports on the plank at A and at B. [4]

(ii) Fig. 3.2 shows the plank horizontal and in equilibrium between a support at C and a peg at D.

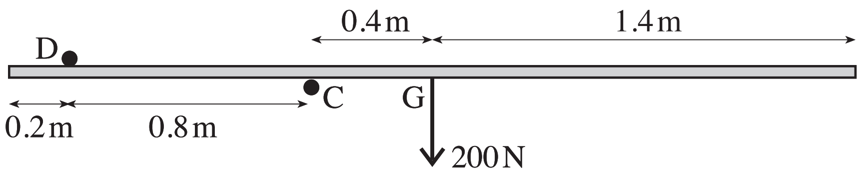

Fig. 3.2

Calculate the reactions of the support and the peg on the plank at C and at D, showing the directions of these forces on a diagram. [5]

Fig. 3.3 shows the plank in equilibrium between a support at P and a peg at Q. The plank is inclined at α to the horizontal, where $\sin\alpha = 0.28$ and $\cos\alpha = 0.96$.

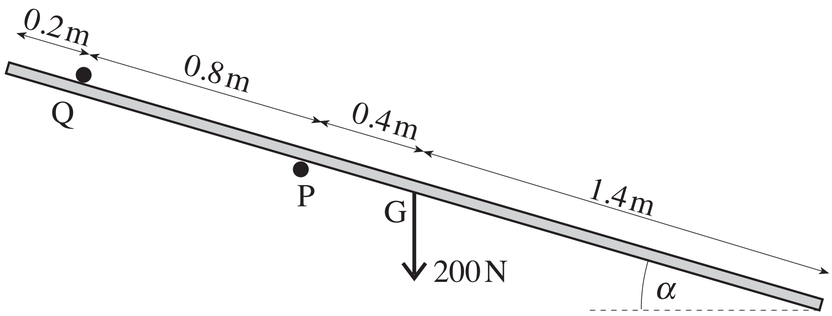

Fig. 3.3

(iii) Calculate the normal reactions at P and at Q. [6]

(iv) Just one of the contacts is rough. Determine which one it is if the value of the coefficient of friction is as small as possible. Find this value of the coefficient of friction. [4]

4 Jack and Jill are raising a pail of water vertically using a light inextensible rope. The pail and water have total mass 20 kg.

In parts **(i)** and **(ii)**, all non-gravitational resistances to motion may be neglected.

(i) How much work is done to raise the pail from rest so that it is travelling upwards at $0.5\,\mathrm{m\,s^{-1}}$ when at a distance of 4 m above its starting position? [4]

(ii) What power is required to raise the pail at a steady speed of $0.5\,\mathrm{m\,s^{-1}}$? [3]

Jack falls over and hurts himself. He then slides down a hill.

His mass is 35 kg and his speed increases from $1\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$ while descending through a vertical height of 3 m.

(iii) How much work is done against friction? [5]

In Jack's further motion, he slides down a slope at an angle α to the horizontal where $\sin\alpha = 0.1$. The frictional force on him is now constant at 150 N. For this part of the motion, Jack's initial speed is $3\,\mathrm{m\,s^{-1}}$.

(iv) How much further does he slide before coming to rest? [5]

OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE **4762/01**

MATHEMATICS (MEI)

Mechanics 2

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

[Turn over

1 **(a)** A battering-ram consists of a wooden beam fixed to a trolley. The battering-ram runs along horizontal ground and collides directly with a vertical wall, as shown in Fig. 1.1. The battering-ram has a mass of 4000 kg.

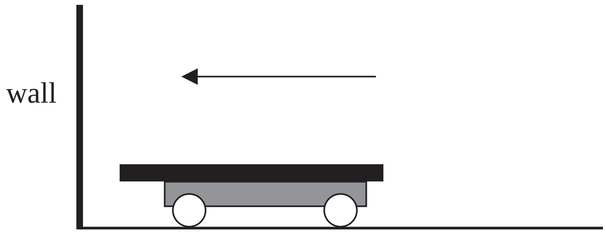

Fig. 1.1

Initially the battering-ram is at rest. Some men push it for 8 seconds and let go just as it is about to hit the wall. While the battering-ram is being pushed, the constant overall force on it in the direction of its motion is 1500 N.

(i) At what speed does the battering-ram hit the wall? [3]

The battering-ram hits a loose stone block of mass 500 kg in the wall. Linear momentum is conserved and the coefficient of restitution in the impact is 0.2.

(ii) Calculate the speeds of the stone block and of the battering-ram immediately after the impact. [6]

(iii) Calculate the energy lost in the impact. [3]

(b) Small objects A and B are sliding on smooth, horizontal ice. Object A has mass 4 kg and speed 18 m s^{-1} in the **i** direction. B has mass 8 kg and speed 9 m s^{-1} in the direction shown in Fig. 1.2, where **i** and **j** are the standard unit vectors.

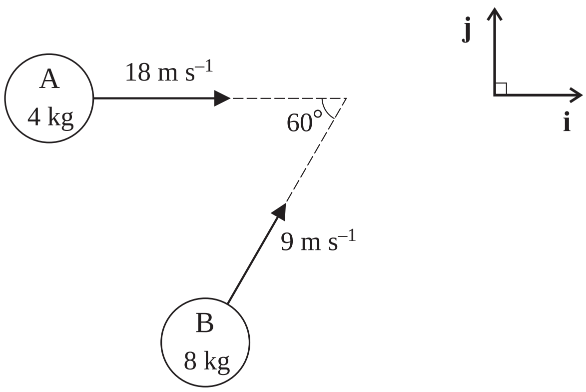

Fig. 1.2

(i) Write down the linear momentum of A and show that the linear momentum of B is $(36\mathbf{i} + 36\sqrt{3}\,\mathbf{j})$ N s. [2]

After the objects meet they stick together (coalesce) and move with a common velocity of $(u\mathbf{i} + v\mathbf{j})\text{ m s}^{-1}$.

(ii) Calculate u and v. [3]

(iii) Find the angle between the direction of motion of the combined object and the **i** direction. Make your method clear. [2]

2 A cyclist and her bicycle have a combined mass of 80 kg.

(i) Initially, the cyclist accelerates from rest to $3\,\text{m}\,\text{s}^{-1}$ against negligible resistances along a horizontal road.

(A) How much energy is gained by the cyclist and bicycle? [2]

(B) The cyclist travels 12 m during this acceleration. What is the average driving force on the bicycle? [2]

(ii) While exerting no driving force, the cyclist free-wheels down a hill. Her speed increases from $4\,\text{m}\,\text{s}^{-1}$ to $10\,\text{m}\,\text{s}^{-1}$. During this motion, the total work done against friction is 1600 J and the drop in vertical height is h m.

Without assuming that the hill is uniform in either its angle or roughness, calculate h. [5]

(iii) The cyclist reaches another horizontal stretch of road and there is now a constant resistance to motion of 40 N.

(A) When the power of the driving force on the bicycle is a constant 200 W, what constant speed can the cyclist maintain? [3]

(B) Find the power of the driving force on the bicycle when travelling at a speed of $0.5\,\text{m}\,\text{s}^{-1}$ with an acceleration of $2\,\text{m}\,\text{s}^{-2}$. [5]

3

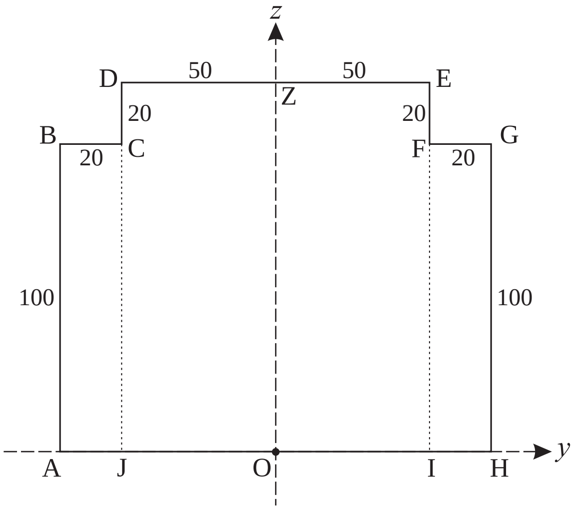

Fig. 3.1

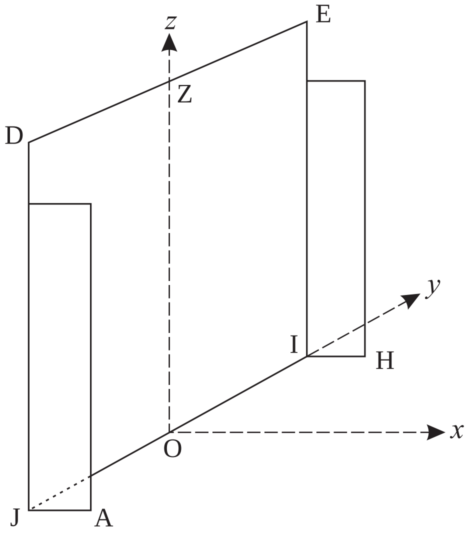

Fig. 3.2

A lamina is made from uniform material in the shape shown in Fig. 3.1. BCJA, DZOJ, ZEIO and FGHI are all rectangles. The lengths of the sides are shown in centimetres.

(i) Find the coordinates of the centre of mass of the lamina, referred to the axes shown in Fig. 3.1. [5]

The rectangles BCJA and FGHI are folded through 90° about the lines CJ and FI respectively to give the fire-screen shown in Fig. 3.2.

(ii) Show that the coordinates of the centre of mass of the fire-screen, referred to the axes shown in Fig. 3.2, are (2.5, 0, 57.5). [4]

The x- and y-axes are in a horizontal floor. The fire-screen has a weight of 72 N. A horizontal force P N is applied to the fire-screen at the point Z. This force is perpendicular to the line DE in the **positive** x direction. The fire-screen is on the point of tipping about the line AH.

(iii) Calculate the value of P. [5]

The coefficient of friction between the fire-screen and the floor is μ.

(iv) For what values of μ does the fire-screen slide before it tips? [4]

4 Fig. 4.1 shows a uniform beam, CE, of weight 2200 N and length 4.5 m. The beam is freely pivoted on a fixed support at D and is supported at C. The distance CD is 2.75 m.

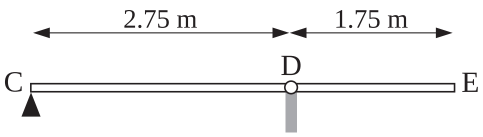

Fig. 4.1

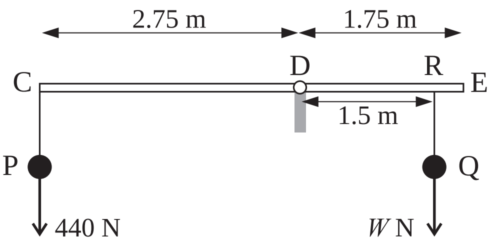

Fig. 4.2

The beam is horizontal and in equilibrium.

(i) Show that the anticlockwise moment of the weight of the beam about D is 1100 N m.

Find the value of the normal reaction on the beam of the support at C. [6]

The support at C is removed and spheres at P and Q are suspended from the beam by light strings attached to the points C and R. The sphere at P has weight 440 N and the sphere at Q has weight W N. The point R of the beam is 1.5 m from D. This situation is shown in Fig. 4.2.

(ii) The beam is horizontal and in equilibrium. Show that $W = 1540$. [3]

The sphere at P is changed for a lighter one with weight 400 N. The sphere at Q is unchanged. The beam is now held in equilibrium at an angle of 20° to the horizontal by means of a light rope attached to the beam at E. This situation (but without the rope at E) is shown in Fig. 4.3.

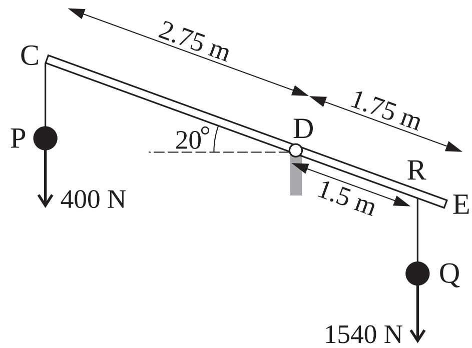

Fig. 4.3

(iii) Calculate the tension in the rope when it is

(A) at 90° to the beam, [6]

(B) horizontal. [3]

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4762/01**

MATHEMATICS (MEI)

Mechanics 2

MONDAY 16 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
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This document consists of **6** printed pages and **2** blank pages.

1 **(a)** Disc A of mass 6 kg and disc B of mass 0.5 kg are moving in the same straight line. The relative positions of the discs and the **i** direction are shown in Fig. 1.1.

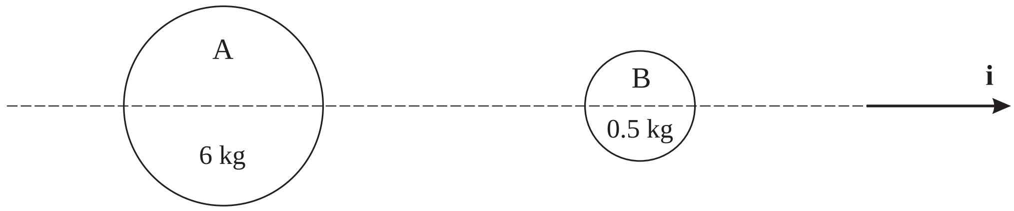

Fig. 1.1

The discs collide directly. The impulse on A in the collision is $-12\mathbf{i}$ N s and after the collision A has velocity $3\mathbf{i}\,\text{m s}^{-1}$ and B has velocity $11\mathbf{i}\,\text{m s}^{-1}$.

(i) Show that the velocity of A just before the collision is $5\mathbf{i}\,\text{m s}^{-1}$ and find the velocity of B at this time. [5]

(ii) Calculate the coefficient of restitution in the collision. [3]

(iii) After the collision, a force of $-2\mathbf{i}$ N acts on B for 7 seconds. Find the velocity of B after this time. [4]

(b) A ball bounces off a smooth plane. The angles its path makes with the plane before and after the impact are α and β, as shown in Fig. 1.2.

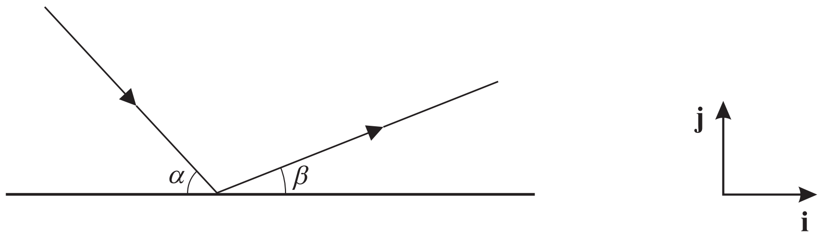

Fig. 1.2

The velocity of the ball before the impact is $u\mathbf{i} - v\mathbf{j}$ and the coefficient of restitution in the impact is e.

Write down an expression in terms of u, v, e, **i** and **j** for the velocity of the ball immediately after the impact. Hence show that $\tan\beta = e\tan\alpha$. [5]

2

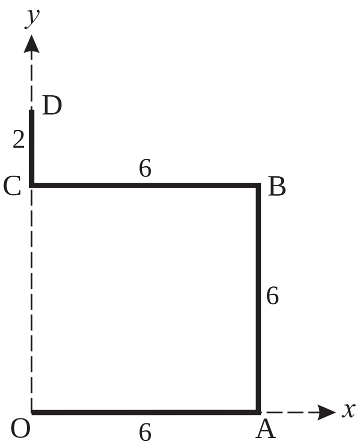

Fig. 2.1

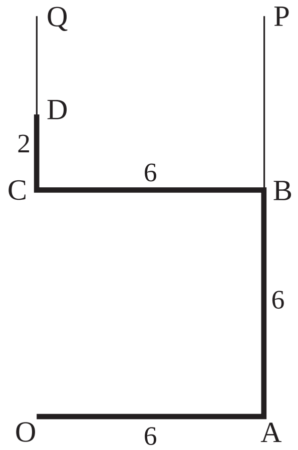

Fig. 2.2

A uniform wire is bent to form a bracket OABCD. The sections OA, AB and BC lie on three sides of a square and CD is parallel to AB. This is shown in Fig. 2.1 where the dimensions, in centimetres, are also given.

(i) Show that, referred to the axes shown in Fig. 2.1, the x-coordinate of the centre of mass of the bracket is 3.6. Find also the y-coordinate of its centre of mass. [6]

(ii) The bracket is now freely suspended from D and hangs in equilibrium.

Draw a diagram showing the position of the centre of mass and calculate the angle of CD to the vertical. [5]

(iii) The bracket is now hung by means of vertical, light strings BP and DQ attached to B and to D, as shown in Fig. 2.2. The bracket has weight 5 N and is in equilibrium with OA horizontal.

Calculate the tensions in the strings BP and DQ. [4]

The original bracket shown in Fig. 2.1 is now changed by adding the section OE, where AOE is a straight line. This section is made of the same type of wire and has length L cm, as shown in Fig. 2.3.

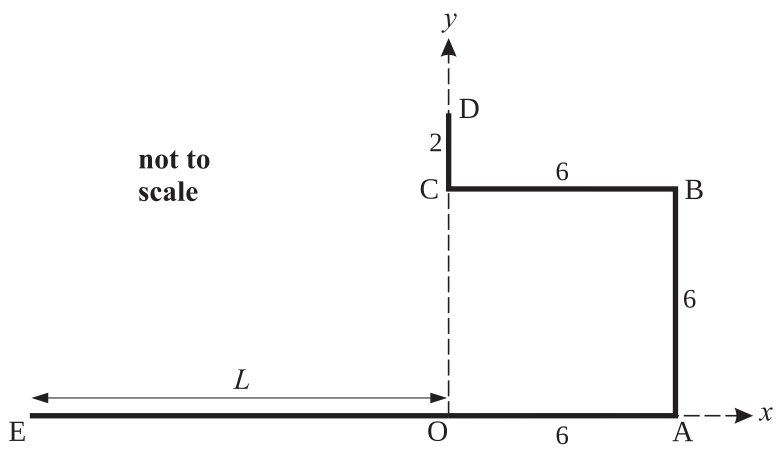

Fig. 2.3

The value of L is chosen so that the centre of mass is now on the y-axis.

(iv) Calculate L. [4]

3 **(a)** Fig. 3.1 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, AD and BD. The rods are freely pin-jointed to each other at A, B and D and to a vertical wall at C and D. There are vertical loads of L N at A and $3L$ N at B. Angle DAB is 30°, angle DBC is 60° and ABC is a straight, horizontal line.

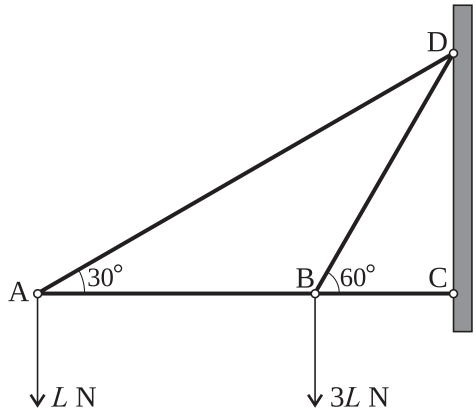

Fig. 3.1

(i) Draw a diagram showing the loads and the internal forces in the four rods. [2]

(ii) Find the internal forces in the rods in terms of L, stating whether each rod is in tension or in thrust (compression). [You may leave answers in surd form. Note that you are **not** required to find the external forces acting at C and at D.] [9]

(b) Fig. 3.2 shows uniform beams PQ and QR, each of length $2l$ m and of weight W N. The beams are freely hinged at Q and are in equilibrium on a rough horizontal surface when inclined at 60° to the horizontal. You are given that the total force acting at Q on QR due to the hinge is horizontal. This force, U N, is shown in Fig. 3.3.

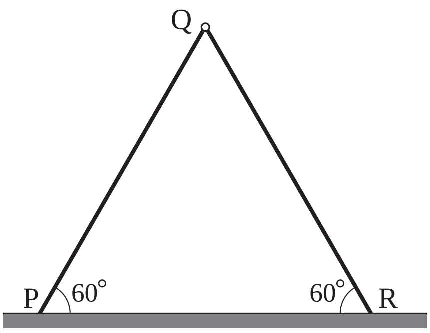

Fig. 3.2

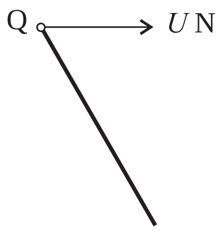

Fig. 3.3

Show that the frictional force between the floor and each beam is $\frac{\sqrt{3}}{6}W$ N. [7]

4 **(a)**

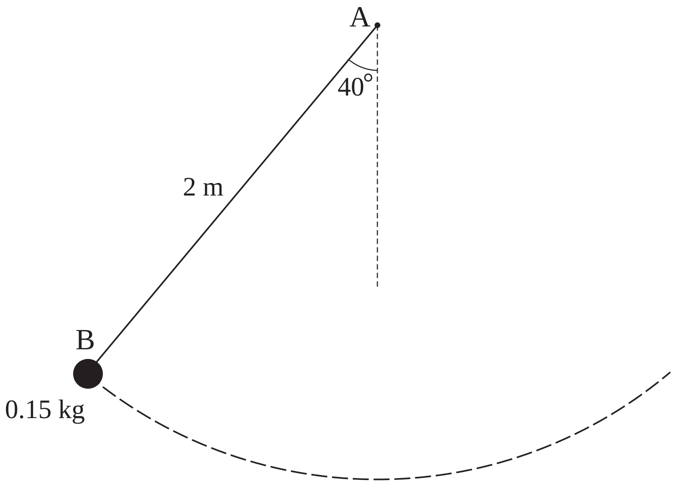

Fig. 4

A small sphere of mass 0.15 kg is attached to one end, B, of a light, inextensible piece of fishing line of length 2 m. The other end of the line, A, is fixed and the line can swing freely.

The sphere swings with the line taut from a point where the line is at an angle of 40° with the vertical, as shown in Fig. 4.

(i) Explain why no work is done on the sphere by the tension in the line. [1]

(ii) Show that the sphere has dropped a vertical distance of about 0.4679 m when it is at the lowest point of its swing and calculate the amount of gravitational potential energy lost when it is at this point. [4]

(iii) Assuming that there is no air resistance and that the sphere swings from rest from the position shown in Fig. 4, calculate the speed of the sphere at the lowest point of its swing. [2]

(iv) Now consider the case where

- there is a force opposing the motion that results in an energy loss of 0.6 J for every metre travelled by the sphere,
- the sphere is given an initial speed of $2.5\,\text{m}\,\text{s}^{-1}$ (and it is descending) with AB at 40° to the vertical.

Calculate the speed of the sphere at the lowest point of its swing. [5]

(b) A block of mass 3 kg slides down a uniform, rough slope that is at an angle of 30° to the horizontal. The acceleration of the block is $\frac{1}{8}g$.

Show that the coefficient of friction between the block and the slope is $\frac{1}{4}\sqrt{3}$. [6]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4762**

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 6 2 *

INSTRUCTIONS TO CANDIDATES

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- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- This document consists of **8** pages. Any blank pages are indicated.

1 **(i)** What constant force is required to accelerate a particle of mass m kg from rest to $2u\,\text{m s}^{-1}$ in 5 seconds? **[3]**

Two discs P and Q are moving in the same straight line over a smooth, horizontal surface. Fig. 1 shows the masses (in kg) and the velocities (in m s^{-1}) of the discs before and after they collide directly. The collision is perfectly elastic.

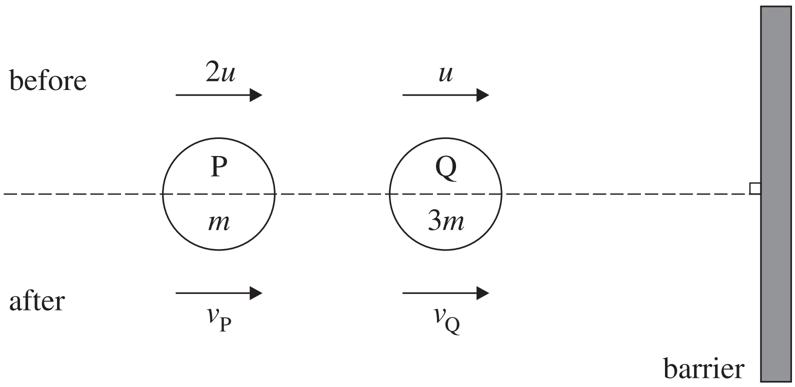

Fig. 1

(ii) Show that $v_Q = \frac{3}{2}u$ and find the value of v_P. **[6]**

After the collision, Q hits the barrier shown in Fig. 1 which is perpendicular to its path and bounces back directly. The coefficient of restitution in this collision is e. Q collides again with P and on this occasion they stick together (coalesce) to form a single object, R, that has a speed of $\frac{1}{4}u\,\text{m s}^{-1}$ away from the barrier.

(iii) Write down an expression in terms of e and u for the velocity of Q after collision with the barrier. Find, in either order, the value of e and the impulse on the barrier. **[9]**

2 One way to load a box into a van is to push the box so that it slides up a ramp. Some removal men are experimenting with the use of different ramps to load a box of mass 80 kg.

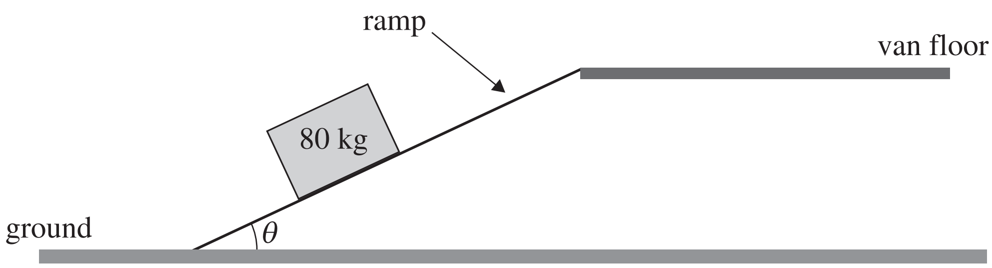

Fig. 2

Fig. 2 shows the general situation. The ramps are all uniformly rough with coefficient of friction 0.4 between the ramp and the box. The men push parallel to the ramp. As the box moves from one end of the ramp to the other it travels a vertical distance of 1.25 m.

(i) Find the limiting frictional force between the ramp and the box in terms of θ. **[3]**

(ii) From rest at the bottom, the box is pushed up the ramp and left at rest at the top. Show that the work done against friction is $\frac{392}{\tan\theta}$ J. **[3]**

(iii) Calculate the gain in the gravitational potential energy of the box when it is raised from the ground to the floor of the van. **[2]**

For the rest of the question take $\theta = 35°$.

(iv) Calculate the power required to slide the box up the ramp at a steady speed of $1.5\,\text{m s}^{-1}$. **[4]**

(v) The box is given an initial speed of $0.5\,\text{m s}^{-1}$ at the top of the ramp and then slides down without anyone pushing it. Determine whether it reaches a speed of $3\,\text{m s}^{-1}$ while it is on the ramp. **[5]**

3 A fish slice consists of a blade and a handle as shown in Fig. 3.1. The rectangular blade ABCD is of mass 250 g and modelled as a lamina; this is 24 cm by 8 cm and is shown in the Oxy plane. The handle EF is of mass 125 g and is modelled as a thin rod; this is 30 cm long and E is attached to the mid-point of CD. EF is at right angles to CD and inclined at α to the plane containing ABCD, where $\sin\alpha = 0.6$ (and $\cos\alpha = 0.8$). Coordinates refer to the axes shown in Fig. 3.1. Lengths are in centimetres. The y- and z-coordinates of the centre of mass of the fish slice are $\bar{y}$ and $\bar{z}$ respectively.

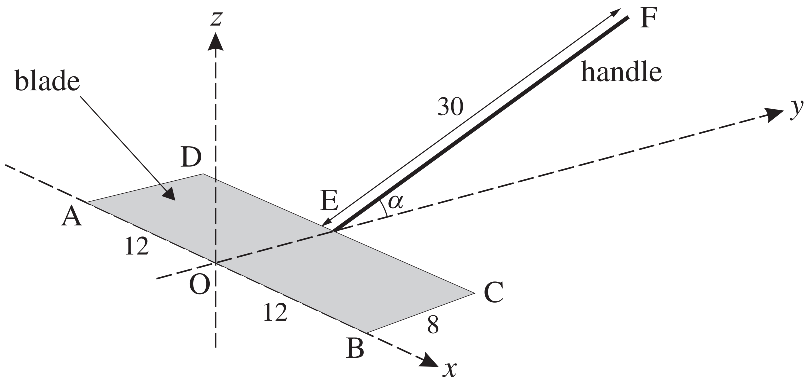

Fig. 3.1

(i) Show that $\bar{y} = 9\frac{1}{3}$ and $\bar{z} = 3$. **[8]**

(ii) Suppose that the plane Oxy in Fig. 3.1 is horizontal and represents a table top and that the fish slice is placed on it as shown. Determine whether the fish slice topples. **[2]**

The 'superior' version of the fish slice has an extra mass of 125 g uniformly distributed over the existing handle for 10 cm from F towards E, as shown in Fig. 3.2. This section of the handle may still be modelled as a thin rod.

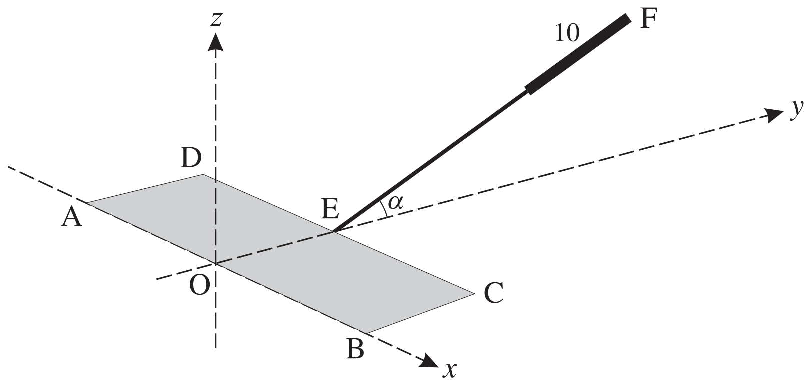

Fig. 3.2

(iii) In this new situation show that $\bar{y} = 14$ and $\bar{z} = 6$. **[4]**

A sales feature of the 'superior' version is the ability to suspend it using a very small hole in the blade. This situation is modelled as the fish slice hanging in equilibrium when suspended freely about an axis through O.

(iv) Indicate the position of the centre of mass on a diagram and calculate the angle of the line OE with the vertical. **[4]**

4 **(a)** A uniform, rigid beam, AB, has a weight of 600 N. It is horizontal and in equilibrium resting on two small smooth pegs at P and Q. Fig. 4.1 shows the positions of the pegs; lengths are in metres.

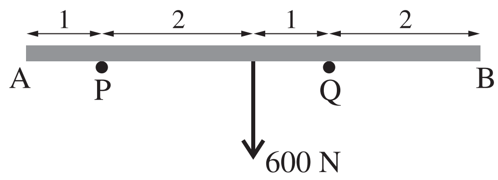

Fig. 4.1

(i) Calculate the forces exerted by the pegs on the beam. **[4]**

A force of L N is applied vertically downwards at B. The beam is in equilibrium but is now on the point of tipping.

(ii) Calculate the value of L. **[3]**

(b) Fig. 4.2 shows a framework in a vertical plane constructed of light, rigid rods AB, BC and CA. The rods are freely pin-jointed to each other at A, B and C and to a fixed point at A. The pin-joint at C rests on a smooth, horizontal support. The dimensions of the framework are shown in metres. There is a force of 340 N acting at B in the plane of the framework. This force and the rod BC are both inclined to the vertical at an angle α, which is defined in triangle BCX. The force on the framework exerted by the support at C is R N.

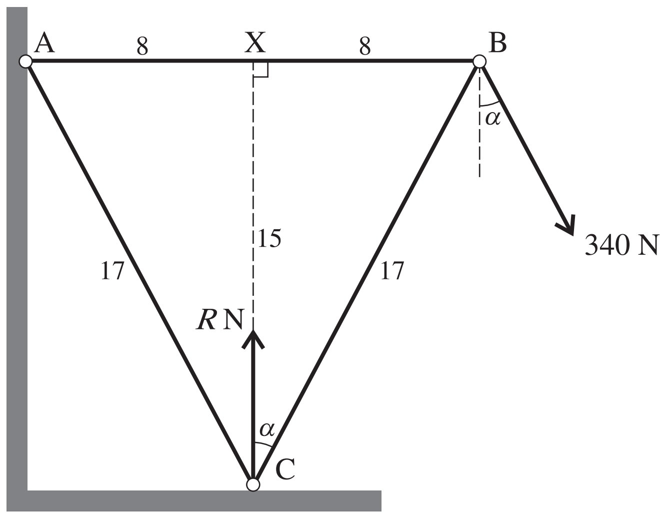

Fig. 4.2

(i) Show that $R = 600$. **[4]**

(ii) Draw a diagram showing all the forces acting on the framework and also the internal forces in the rods. **[2]**

(iii) Calculate the internal forces in the three rods, indicating whether each rod is in tension or in compression (thrust). [Your working in this part should correspond to your diagram in part **(ii)**.] **[6]**

ADVANCED GCE

MATHEMATICS (MEI) 4762

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
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Turn over

1 **(a)** Two small objects, P of mass m kg and Q of mass km kg, slide on a smooth horizontal plane. Initially, P and Q are moving in the same straight line towards one another, each with speed $u \text{ m s}^{-1}$.

After a direct collision with P, the direction of motion of Q is reversed and it now has a speed of $\frac{1}{3}u \text{ m s}^{-1}$. The velocity of P is now $v \text{ m s}^{-1}$, where the positive direction is the original direction of motion of P.

(i) Draw a diagram showing the velocities of P and Q before and after the impact. **[1]**

(ii) By considering the linear momentum of the objects before and after the collision, show that $v = \left(1 - \frac{4}{3}k\right)u$. **[3]**

(iii) Hence find the condition on k for the direction of motion of P to be reversed. **[2]**

The coefficient of restitution in the collision is 0.5.

(iv) Show that $v = -\frac{2}{3}u$ and calculate the value of k. **[5]**

(b) Particle A has a mass of 5 kg and velocity $\begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ m s}^{-1}$. Particle B has mass 3 kg and is initially at rest. A force $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ N acts for 9 seconds on B and subsequently (in the absence of the force), A and B collide and stick together to form an object C that moves off with a velocity $\mathbf{V} \text{ m s}^{-1}$.

(i) Show that $\mathbf{V} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. **[4]**

The object C now collides with a smooth barrier which lies in the direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The coefficient of restitution in the collision is 0.5.

(ii) Calculate the velocity of C after the impact. **[3]**

2 **(a)** A small block of mass 25 kg is on a long, horizontal table. Each side of the block is connected to a small sphere by means of a light inextensible string passing over a smooth pulley. Fig. 2 shows this situation. Sphere A has mass 5 kg and sphere B has mass 20 kg. Each of the spheres hangs freely.

Fig. 2

Initially the block moves on a smooth part of the table. With the block at a point O, the system is released from rest with both strings taut.

(i) (*A*) Is mechanical energy conserved in the subsequent motion? Give a brief reason for your answer. **[1]**

(*B*) Why is no work done by the block against the reaction of the table on it? **[1]**

The block reaches a speed of $1.5\ \text{m s}^{-1}$ at point P.

(ii) Use an energy method to calculate the distance OP. **[5]**

The block continues moving beyond P, at which point the table becomes rough. After travelling two metres beyond P, the block passes through point Q. The block does 180 J of work against resistances to its motion from P to Q.

(iii) Use an energy method to calculate the speed of the block at Q. **[5]**

(b) A tree trunk of mass 450 kg is being pulled up a slope inclined at 20° to the horizontal.

Calculate the power required to pull the trunk at a steady speed of $2.5\ \text{m s}^{-1}$ against a frictional force of 2000 N. **[5]**

3 A non-uniform beam AB has weight 85 N. The length of the beam is 5 m and its centre of mass is 3 m from A. In this question all the forces act in the same vertical plane.

Fig. 3.1 shows the beam in horizontal equilibrium, supported at its ends.

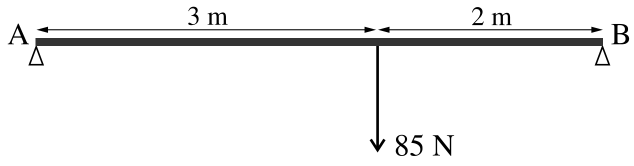

Fig. 3.1

(i) Calculate the reactions of the supports on the beam. **[4]**

Using a smooth hinge, the beam is now attached at A to a vertical wall. The beam is held in equilibrium at an angle α to the horizontal by means of a horizontal force of magnitude 27.2 N acting at B. This situation is shown in Fig. 3.2.

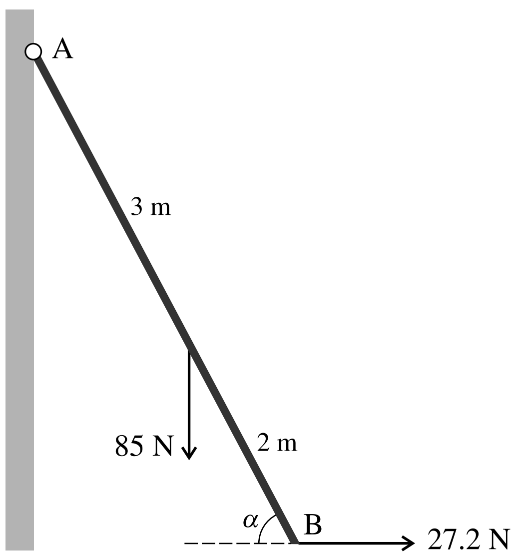

Fig. 3.2

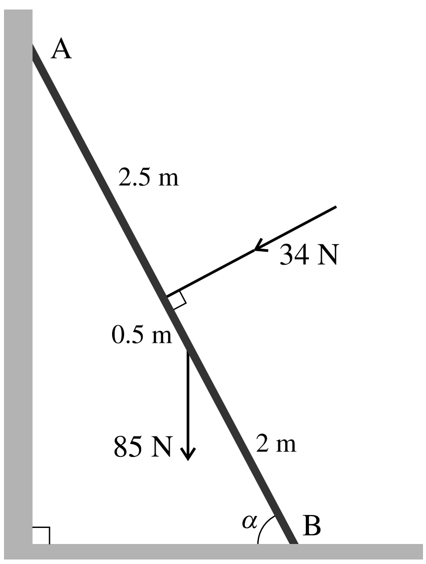

Fig. 3.3

(ii) Show that $\tan \alpha = \frac{15}{8}$. **[4]**

The hinge and 27.2 N force are removed. The beam now rests with B on a rough horizontal floor and A on a smooth vertical wall, as shown in Fig. 3.3. It is at the same angle α to the horizontal. There is now a force of 34 N acting at right angles to the beam at its centre in the direction shown. The beam is in equilibrium and on the point of slipping.

(iii) Draw a diagram showing the forces acting on the beam.

Show that the frictional force acting on the beam is 7.4 N.

Calculate the value of the coefficient of friction between the beam and the floor. **[10]**

4 In this question you may use the following facts: as illustrated in Fig. 4.1, the centre of mass, G, of a uniform thin open hemispherical shell is at the mid-point of OA on its axis of symmetry; the surface area of this shell is $2\pi r^2$, where r is the distance OA.

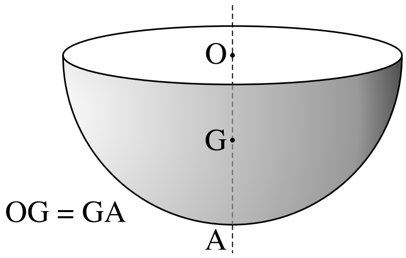

Fig. 4.1

A perspective view and a cross-section of a dog bowl are shown in Fig. 4.2. The bowl is made throughout from thin uniform material. An open hemispherical shell of radius 8 cm is fitted inside an open circular cylinder of radius 8 cm so that they have a common axis of symmetry and the rim of the hemisphere is at one end of the cylinder. The height of the cylinder is k cm. The point O is on the axis of symmetry and at the end of the cylinder.

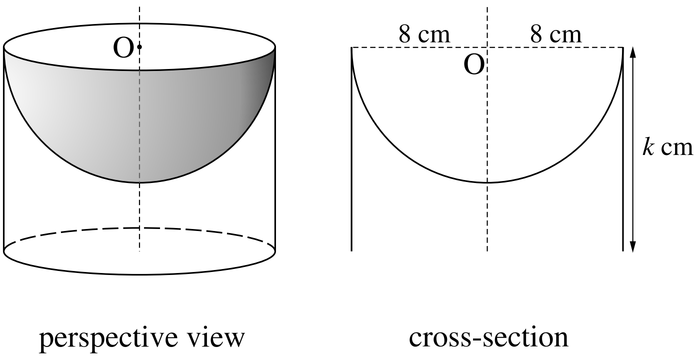

Fig. 4.2

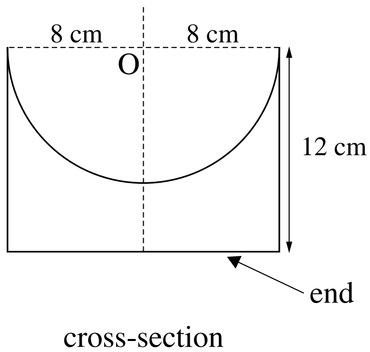

Fig. 4.3

(i) Show that the centre of mass of the bowl is a distance $\dfrac{64 + k^2}{16 + 2k}$ cm from O. **[6]**

A version of the bowl for the 'senior dog' has $k = 12$ and an end to the cylinder, as shown in Fig. 4.3. The end is made from the same material as the original bowl.

(ii) Show that the centre of mass of this bowl is a distance $6\frac{1}{3}$ cm from O. **[5]**

This bowl is placed on a rough slope inclined at θ to the horizontal.

(iii) Assume that the bowl is prevented from sliding and is on the point of toppling.

Draw a diagram indicating the position of the centre of mass of the bowl with relevant lengths marked.

Calculate the value of θ. **[5]**

(iv) If the bowl is not prevented from sliding, determine whether it will slide when placed on the slope when there is a coefficient of friction between the bowl and the slope of 1.5. **[3]**

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MATHEMATICS (MEI) **4762**

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

4762

INSTRUCTIONS TO CANDIDATES

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1 **(a)** An object P, with mass 6 kg and speed $1\ \text{m s}^{-1}$, is sliding on a smooth horizontal table. Object P explodes into two small parts, Q and R. Q has mass 4 kg and R has mass 2 kg and speed $4\ \text{m s}^{-1}$ in the direction of motion of P before the explosion. This information is shown in Fig. 1.1.

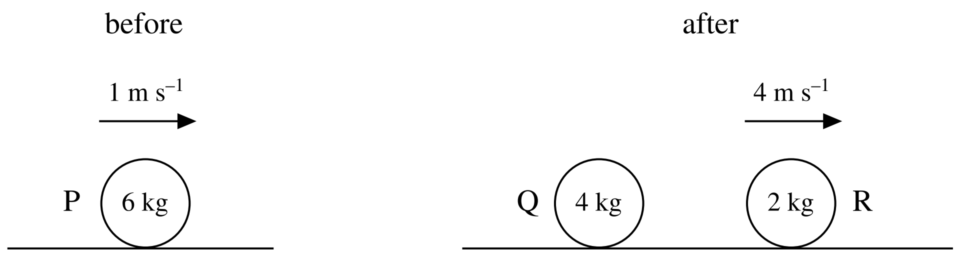

Fig. 1.1

(i) Calculate the velocity of Q. **[4]**

Just as object R reaches the edge of the table, it collides directly with a small object S of mass 3 kg that is travelling horizontally towards R with a speed of $1\ \text{m s}^{-1}$. This information is shown in Fig. 1.2. The coefficient of restitution in this collision is 0.1.

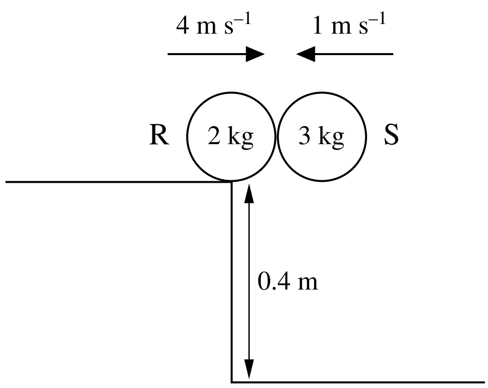

Fig. 1.2

(ii) Calculate the velocities of R and S immediately after the collision. **[6]**

The table is 0.4 m above a horizontal floor. After the collision, R and S have no contact with the table.

(iii) Calculate the distance apart of R and S when they reach the floor. **[3]**

(b) A particle of mass m kg bounces off a smooth horizontal plane. The components of velocity of the particle just before the impact are $u\ \text{m s}^{-1}$ parallel to the plane and $v\ \text{m s}^{-1}$ perpendicular to the plane. The coefficient of restitution is e.

Show that the mechanical energy lost in the impact is $\frac{1}{2}mv^2(1 - e^2)$ J. **[4]**

2 A car of mass 1200 kg travels along a road for two minutes during which time it rises a vertical distance of 60 m and does 1.8×10^6 J of work against the resistance to its motion. The speeds of the car at the start and at the end of the two minutes are the same.

(i) Calculate the average power developed over the two minutes. **[4]**

The car now travels along a straight level road at a steady speed of $18\ \text{m s}^{-1}$ while developing constant power of 13.5 kW.

(ii) Calculate the resistance to the motion of the car.

How much work is done against the resistance when the car travels 200 m? **[5]**

While travelling at $18\ \text{m s}^{-1}$, the car starts to go **down** a slope inclined at $5°$ to the horizontal with the power removed and its brakes applied. The total resistance to its motion is now 1500 N.

(iii) Use an energy method to determine how far down the slope the car travels before its speed is halved. **[6]**

Suppose the car is travelling along a straight level road and developing power P W while travelling at $v\ \text{m s}^{-1}$ with acceleration $a\ \text{m s}^{-2}$ against a resistance of R N.

(iv) Show that $P = (R + 1200a)v$ and deduce that if P and R are constant then if a is not zero it cannot be constant. **[4]**

3 A side view of a free-standing kitchen cupboard on a horizontal floor is shown in Fig. 3.1. The cupboard consists of: a base CE; vertical ends BC and DE; an overhanging horizontal top AD. The dimensions, in metres, of the cupboard are shown in the figure. The cupboard and contents have a weight of 340 N and centre of mass at G.

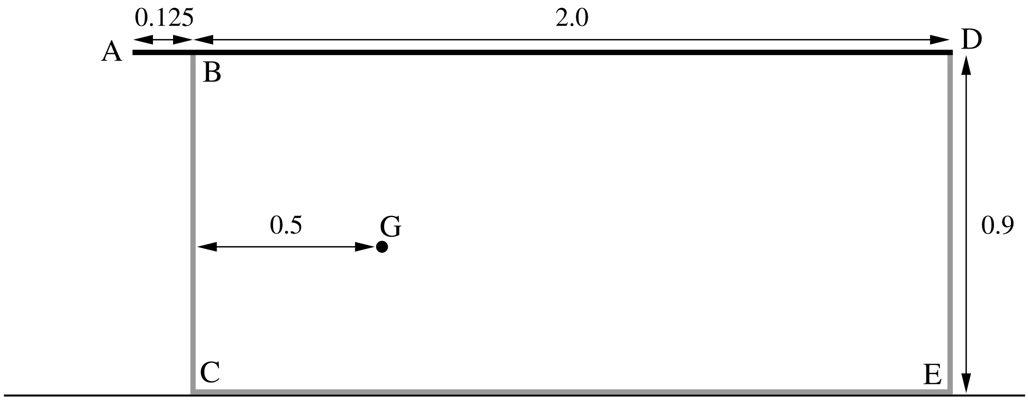

Fig. 3.1

(i) Calculate the magnitude of the vertical force required at A for the cupboard to be on the point of tipping in the cases where the force acts

(*A*) downwards, **[3]**

(*B*) upwards. **[3]**

A force of magnitude Q N is now applied at A at an angle of θ to AB, as shown in Fig. 3.2, where $\cos\theta = \frac{5}{13}$ (and $\sin\theta = \frac{12}{13}$).

Fig. 3.2

(ii) By considering the vertical and horizontal components of the force at A, show that the clockwise moment of this force about E is $\frac{30Q}{13}$ N m. **[3]**

With the force of magnitude Q N acting as shown in Fig. 3.2, the cupboard is in equilibrium and is on the point of tipping but not on the point of sliding.

(iii) Show that $Q = 221$ and that the coefficient of friction between the cupboard base and the floor must be greater than $\frac{5}{8}$. **[9]**

4 In this question, coordinates refer to the axes shown in the figures and the units are centimetres.

Fig. 4.1 shows a lamina KLMNOP shaded. The lamina is made from uniform material and has the dimensions shown.

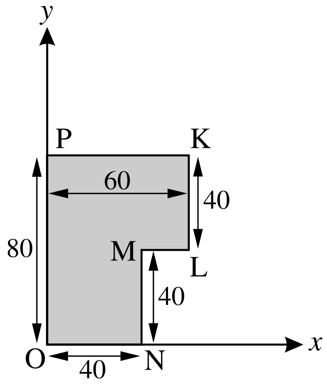

Fig. 4.1

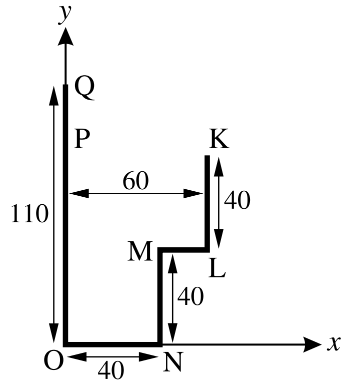

Fig. 4.2

(i) Show that the x-coordinate of the centre of mass of this lamina is 26 and calculate the y-coordinate. **[4]**

A uniform thin heavy wire KLMNOPQ is bent into the shape of part of the perimeter of the lamina KLMNOP with an extension of the side OP to Q, as shown in Fig. 4.2.

(ii) Show that the x-coordinate of the centre of mass of this wire is 23.2 and calculate the y-coordinate. **[5]**

The wire is freely suspended from Q and hangs in equilibrium.

(iii) Draw a diagram indicating the position of the centre of mass of the hanging wire and calculate the angle of QO with the vertical. **[4]**

A wall-mounted bin with an open top is shown in Fig. 4.3. The centre part has cross-section KLMNOPQ; the two ends are in the shape of the lamina KLMNOP.

The ends are made from the same uniform, thin material and each has a mass of 1.5 kg. The centre part is made from different uniform, thin material and has a total mass of 7 kg.

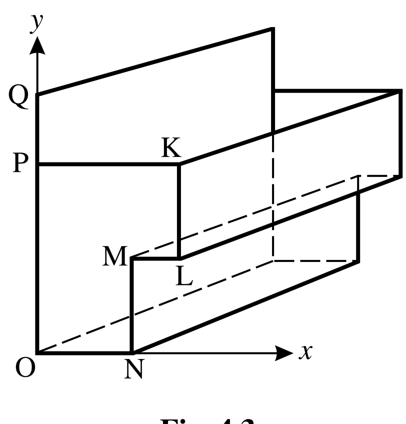

Fig. 4.3

(iv) Calculate the x- and y-coordinates of the centre of mass of the bin. **[5]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4762**

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 6 2 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 Two sledges P and Q, with their loads, have masses of 200 kg and 250 kg respectively and are sliding on a horizontal surface against negligible resistance. There is an inextensible light rope connecting the sledges that is horizontal and parallel to the direction of motion.

Fig. 1 shows the initial situation with both sledges travelling with a velocity of $5\mathbf{i}\,\text{m s}^{-1}$.

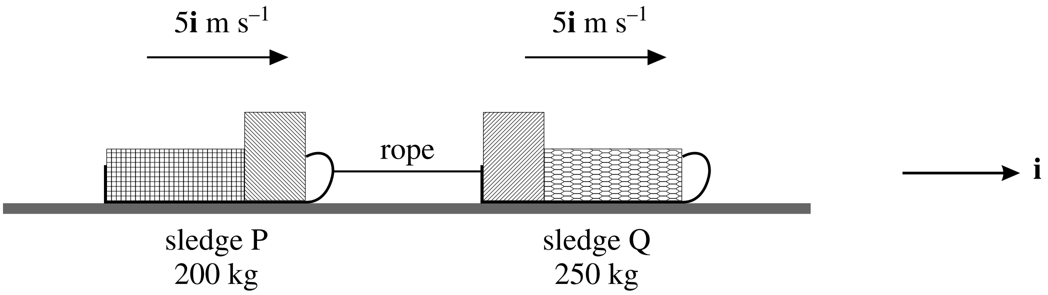

Fig. 1

A mechanism on Q jerks the rope so that there is an impulse of $250\mathbf{i}$ N s on P.

(i) Show that the new velocity of P is $6.25\mathbf{i}\,\text{m s}^{-1}$ and find the new velocity of Q. **[4]**

There is now a direct collision between the sledges and after the impact P has velocity $4.5\mathbf{i}\,\text{m s}^{-1}$.

(ii) Show that the velocity of Q becomes $5.4\mathbf{i}\,\text{m s}^{-1}$.

Calculate the coefficient of restitution in the collision. **[6]**

Before the rope becomes taut again, the velocity of P is increased so that it catches up with Q. This is done by throwing part of the load from sledge P in the $-\mathbf{i}$ direction so that P's velocity increases to $5.5\mathbf{i}\,\text{m s}^{-1}$. The part of the load thrown out is an object of mass 20 kg.

(iii) Calculate the speed of separation of the object from P. **[5]**

When the sledges directly collide again they are held together and move as a single object.

(iv) Calculate the common velocity of the pair of sledges, giving your answer correct to 3 significant figures. **[2]**

2

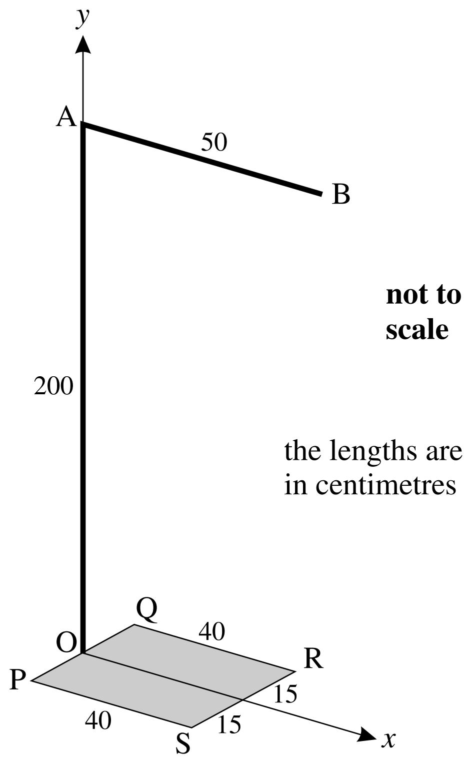

Fig. 2

Fig. 2 shows a stand on a horizontal floor and horizontal and vertical coordinate axes Ox and Oy. The stand is modelled as

- a thin uniform rectangular base PQRS, 30 cm by 40 cm with mass 15 kg; the sides QR and PS are parallel to Ox,
- a thin uniform vertical rod of length 200 cm and mass 3 kg that is fixed to the base at O, the mid-point of PQ and the origin of coordinates,
- a thin uniform top rod AB of length 50 cm and mass 2 kg; AB is parallel to Ox.

Coordinates are referred to the axes shown in the figure.

(i) Calculate the coordinates of the centre of mass of the stand. **[5]**

A small object of mass 5 kg is fixed to the rod AB at a distance of 40 cm from A.

(ii) Show that the coordinates of the centre of mass of the stand with the object are (22, 68). **[2]**

The stand is tilted about the edge PQ until it is on the point of toppling. The angle through which the stand is tilted is called 'the angle of tilt'. This procedure is repeated about the edges QR and RS.

(iii) Making your method clear, determine which edge requires the smallest angle of tilt for the stand to topple. **[4]**

The small object is removed. A light string is attached to the stand at A and pulled at an angle of 50° to the downward vertical in the plane Oxy in an attempt to tip the stand about the edge RS.

(iv) Assuming that the stand does not slide, find the tension in the string when the stand is about to turn about the edge RS. **[7]**

3 Fig. 3 shows a framework in a vertical plane constructed of light, rigid rods AB, BC, CD, DA and BD. The rods are freely pin-jointed to each other at A, B, C and D and to a vertical wall at A. ABCD is a parallelogram with AD horizontal and BD vertical; the dimensions of the framework, in metres, are shown. There is a vertical load of 300 N acting at C and a vertical wire attached to D, with tension T N, holds the framework in equilibrium. The horizontal and vertical forces, X N and Y N, acting on the framework at A due to the wall are also shown.

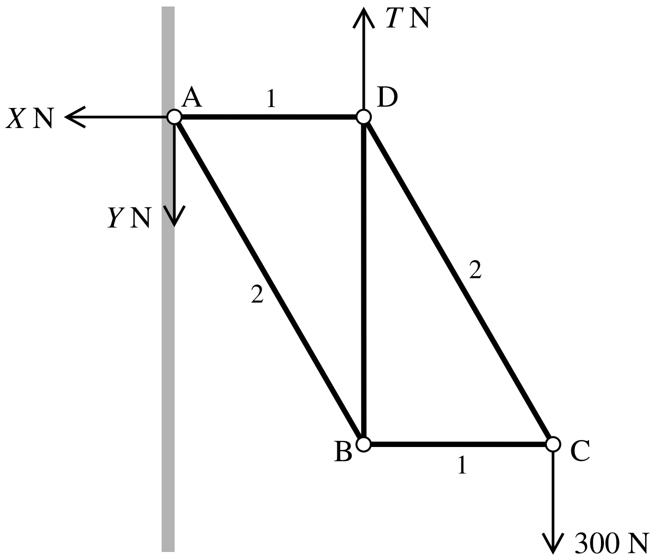

Fig. 3

(i) Show that $T = 600$ and calculate the values of X and Y. **[4]**

(ii) Draw a diagram showing all the forces acting on the framework, and also the internal forces in the rods. **[2]**

(iii) Calculate the internal forces in the five rods, indicating whether each rod is in tension or compression (thrust). (You may leave answers in surd form. Your working in this part should correspond to your diagram in part **(ii)**.) **[9]**

Suppose that the vertical wire is attached at B instead of D and that the framework is still in equilibrium.

(iv) Without doing any further calculations, state which four of the rods have the same internal forces as in part **(iii)** and say briefly why this is the case. Determine the new force in the fifth rod. **[4]**

4 A box of mass 16 kg is on a uniformly rough horizontal floor with an applied force of fixed direction but varying magnitude P N acting as shown in Fig. 4. You may assume that the box does not tip for any value of P. The coefficient of friction between the box and the floor is μ.

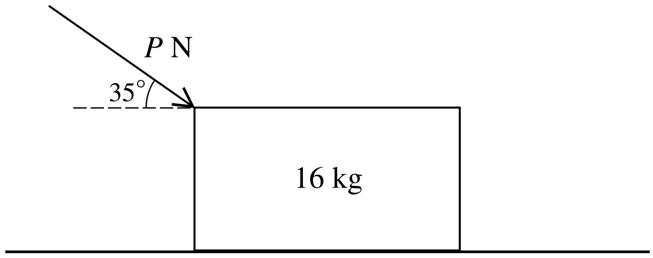

Fig. 4

Initially the box is at rest and on the point of slipping with $P = 58$.

(i) Show that μ is about 0.25. **[5]**

In the rest of this question take μ to be exactly 0.25.

The applied force on the box is suddenly increased so that $P = 70$ and the box moves against friction with the floor and another horizontal retarding force, S. The box reaches a speed of $1.5\,\text{m}\,\text{s}^{-1}$ from rest after 5 seconds; during this time the box slides 3 m.

(ii) Calculate the work done by the applied force of 70 N and also the average power developed by this force over the 5 seconds. **[4]**

(iii) By considering the values of time, distance and speed, show that an object starting from rest that travels 3 m while reaching a speed of $1.5\,\text{m}\,\text{s}^{-1}$ in 5 seconds cannot be moving with constant acceleration. **[2]**

The reason that the acceleration is not constant is that the retarding force S is not constant.

(iv) Calculate the total work done by the retarding force S. **[7]**

ADVANCED GCE

MATHEMATICS (MEI) 4762

Mechanics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4762
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Fig. 1.1 shows block A of mass 2.5 kg which has been placed on a long, uniformly rough slope inclined at an angle α to the horizontal, where $\cos \alpha = 0.8$. The coefficient of friction between A and the slope is 0.85.

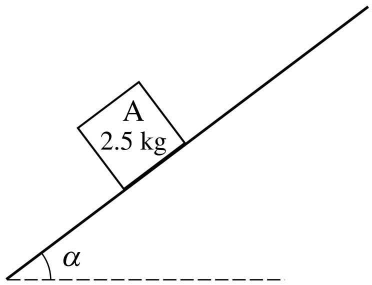

Fig. 1.1

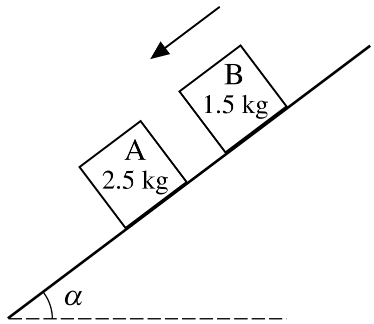

Fig. 1.2

(i) Calculate the maximum possible frictional force between A and the slope.

Show that A will remain at rest. **[6]**

With A still at rest, block B of mass 1.5 kg is projected down the slope, as shown in Fig. 1.2. B has a speed of $16\,\text{m}\,\text{s}^{-1}$ when it collides with A. In this collision the coefficient of restitution is 0.4, the impulses are parallel to the slope and linear momentum parallel to the slope is conserved.

(ii) Show that the velocity of A immediately after the collision is $8.4\,\text{m}\,\text{s}^{-1}$ down the slope.

Find the velocity of B immediately after the collision. **[6]**

(iii) Calculate the impulse on B in the collision. **[3]**

The blocks do not collide again.

(iv) For what length of time after the collision does A slide before it comes to rest? **[4]**

2 **(a)** A firework is instantaneously at rest in the air when it explodes into two parts. One part is the body B of mass 0.06 kg and the other a cap C of mass 0.004 kg. The total kinetic energy given to B and C is 0.8 J. B moves off horizontally in the **i** direction.

By considering both kinetic energy and linear momentum, calculate the velocities of B and C immediately after the explosion. **[8]**

(b) A car of mass 800 kg is travelling up some hills.

In one situation the car climbs a vertical height of 20 m while its speed decreases from $30\,\mathrm{m\,s^{-1}}$ to $12\,\mathrm{m\,s^{-1}}$. The car is subject to a resistance to its motion but there is no driving force and the brakes are not being applied.

(i) Using an energy method, calculate the work done by the car against the resistance to its motion. **[4]**

In another situation the car is travelling at a constant speed of $18\,\mathrm{m\,s^{-1}}$ and climbs a vertical height of 20 m in 25 s up a uniform slope. The resistance to its motion is now 750 N.

(ii) Calculate the power of the driving force required. **[5]**

3

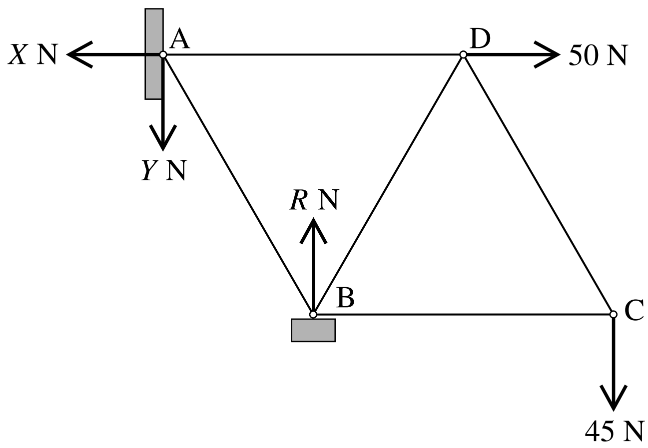

Fig. 3

Fig. 3 shows a framework in equilibrium in a vertical plane. The framework is made from the equal, light, rigid rods AB, AD, BC, BD and CD so that ABD and BCD are equilateral triangles of side 2 m. AD and BC are horizontal.

The rods are freely pin-jointed to each other at A, B, C and D. The pin-joint at A is fixed to a wall and the pin-joint at B rests on a smooth horizontal support.

Fig. 3 also shows the external forces acting on the framework: there is a vertical load of 45 N at C and a horizontal force of 50 N applied at D; the normal reaction of the support on the framework at B is R N; horizontal and vertical forces X N and Y N act at A.

(i) Write down equations for the horizontal and vertical equilibrium of the framework. **[2]**

(ii) Show that $R = 135$ and $Y = 90$. **[3]**

(iii) On the diagram in your printed answer book, show the forces internal to the rods acting on the pin-joints. **[2]**

(iv) Calculate the forces internal to the five rods, stating whether each rod is in tension or compression (thrust). [You may leave your answers in surd form. Your working in this part should correspond to your diagram in part **(iii)**.] **[10]**

(v) Suppose that the force of magnitude 50 N applied at D is no longer horizontal, and the system remains in equilibrium in the same position.

By considering the equilibrium at C, show that the forces in rods CD and BC are not changed. **[2]**

4 You are given that the centre of mass, G, of a uniform lamina in the shape of an isosceles triangle lies on its axis of symmetry in the position shown in Fig. 4.1.

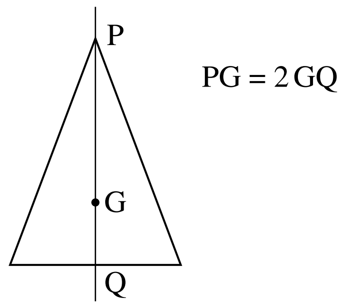

Fig. 4.1

Fig. 4.2 shows the cross-section OABCD of a prism made from uniform material. OAB is an isosceles triangle, where OA = AB, and OBCD is a rectangle. The distance OD is h cm, where h can take various positive values. All coordinates refer to the axes Ox and Oy shown. The units of the axes are centimetres.

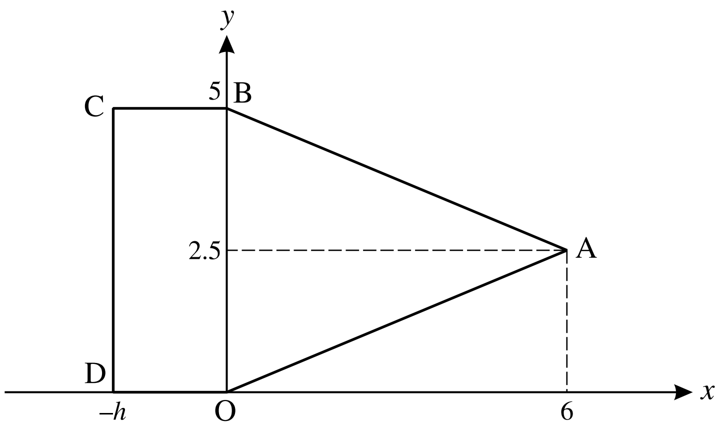

Fig. 4.2

(i) Write down the coordinates of the centre of mass of the triangle OAB. **[1]**

(ii) Show that the centre of mass of the region OABCD is $\left(\dfrac{12-h^2}{2(h+3)}, 2.5\right)$. **[6]**

The x-axis is horizontal.

The prism is placed on a horizontal plane in the position shown in Fig. 4.2.

(iii) Find the values of h for which the prism would topple. **[3]**

The following questions refer to the case where $h = 3$ with the prism held in the position shown in Fig. 4.2. The cross-section OABCD contains the centre of mass of the prism. The weight of the prism is 15 N. You should assume that the prism does not slide.

(iv) Suppose that the prism is held in this position by a vertical force applied at A. Given that the prism is on the point of tipping clockwise, calculate the magnitude of this force. **[3]**

(v) Suppose instead that the prism is held in this position by a force in the plane of the cross-section OABCD, applied at 30° below the horizontal at C, as shown in Fig. 4.3. Given that the prism is on the point of tipping anti-clockwise, calculate the magnitude of this force. **[4]**

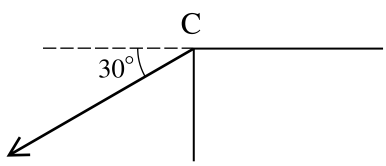

Fig. 4.3

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OCR
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ADVANCED GCE

MATHEMATICS (MEI) **4762**

Mechanics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4762
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 **(a)** Sphere P, of mass 10 kg, and sphere Q, of mass 15 kg, move with their centres on a horizontal straight line and have no resistances to their motion. P, Q and the positive direction are shown in Fig. 1.1.

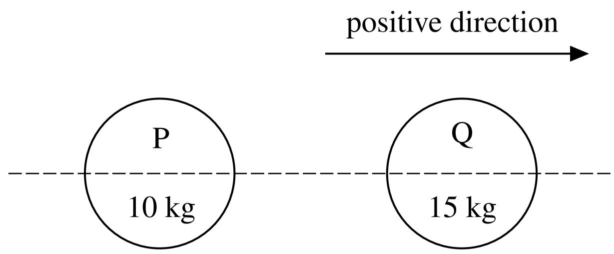

Fig. 1.1

Initially, P has a velocity of $-1.75\,\text{m s}^{-1}$ and is acted on by a force of magnitude 13 N acting in the direction PQ.

After T seconds, P has a velocity of $4.75\,\text{m s}^{-1}$ and has not reached Q.

(i) Calculate T. **[3]**

The force of magnitude 13 N is removed. P is still travelling at $4.75\,\text{m s}^{-1}$ when it collides directly with Q, which has a velocity of $-0.5\,\text{m s}^{-1}$.

Suppose that P and Q coalesce in the collision to form a single object.

(ii) Calculate their common velocity after the collision. **[2]**

Suppose instead that P and Q separate after the collision and that P has a velocity of $1\,\text{m s}^{-1}$ afterwards.

(iii) Calculate the velocity of Q after the collision and also the coefficient of restitution in the collision. **[6]**

(b) Fig. 1.2 shows a small ball projected at a speed of $14\,\text{m s}^{-1}$ at an angle of $30°$ below the horizontal over smooth horizontal ground.

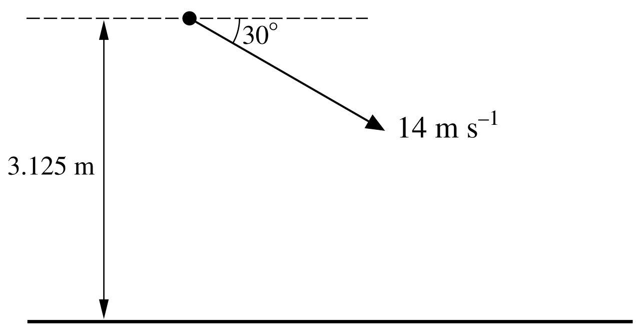

Fig. 1.2

The ball is initially 3.125 m above the ground. The coefficient of restitution between the ball and the ground is 0.6.

Calculate the angle with the horizontal of the ball's trajectory immediately after the **second** bounce on the ground. **[8]**

2 Any non-exact answers to this question should be given correct to four significant figures.

A thin, straight beam AB is 2 m long. It has a weight of 600 N and its centre of mass G is 0.8 m from end A. The beam is freely pivoted about a horizontal axis through A.

The beam is held horizontally in equilibrium.

Initially this is done by means of a support at B, as shown in Fig. 2.1.

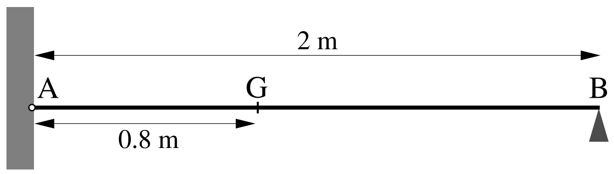

Fig. 2.1

(i) Calculate the force on the beam due to the support at B. **[3]**

The support at B is now removed and replaced by a wire attached to the beam 0.3 m from A and inclined at 50° to the beam, as shown in Fig. 2.2. The beam is still horizontal and in equilibrium.

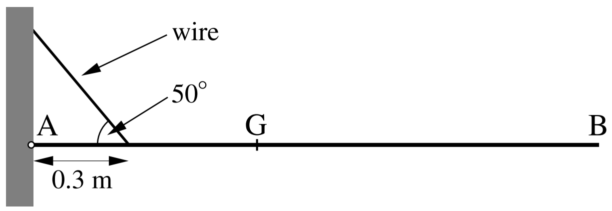

Fig. 2.2

(ii) Calculate the tension in the wire. **[5]**

The forces acting on the beam at A due to the hinge are a horizontal force X N in the direction AB, and a downward vertical force Y N, which have a resultant of magnitude R at α to the horizontal.

(iii) Calculate X, Y, R and α. **[7]**

The dotted lines in Fig. 2.3 are the lines of action of the tension in the wire and the weight of the beam. These lines of action intersect at P.

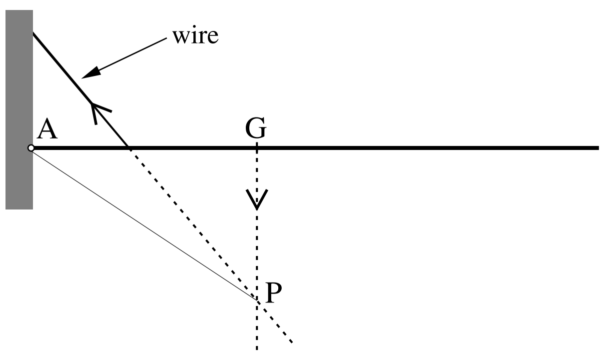

Fig. 2.3

(iv) State with a reason the size of the angle GAP. **[2]**

3 A bracket is being made from a sheet of uniform thin metal. Firstly, a plate is cut from a square of the sheet metal in the shape OABCDEFHJK, shown shaded in Fig. 3.1. The dimensions shown in the figure are in centimetres; axes Ox and Oy are also shown.

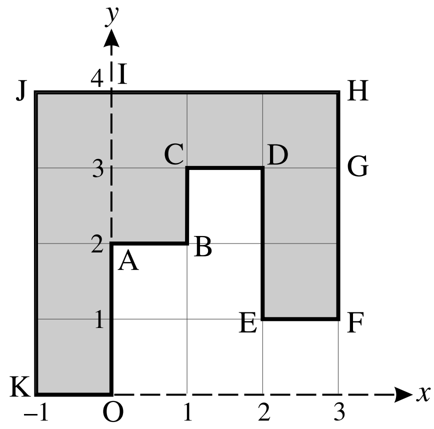

Fig. 3.1

(i) Show that, referred to the axes given in Fig. 3.1, the centre of mass of the plate OABCDEFHJK has coordinates (0.8, 2.5). **[4]**

The plate is hung using light vertical strings attached to J and H. The edge JH is horizontal and the plate is in equilibrium. The weight of the plate is 3.2 N.

(ii) Calculate the tensions in each of the strings. **[5]**

The plate is now bent to form the bracket. This is shown in Fig. 3.2: the rectangle IJKO is folded along the line IA so that it is perpendicular to the plane ABCGHI; the rectangle DEFG is folded along the line DG so it is also perpendicular to the plane ABCGHI but on the other side of it. Fig. 3.2 also shows the axes Ox, Oy and Oz.

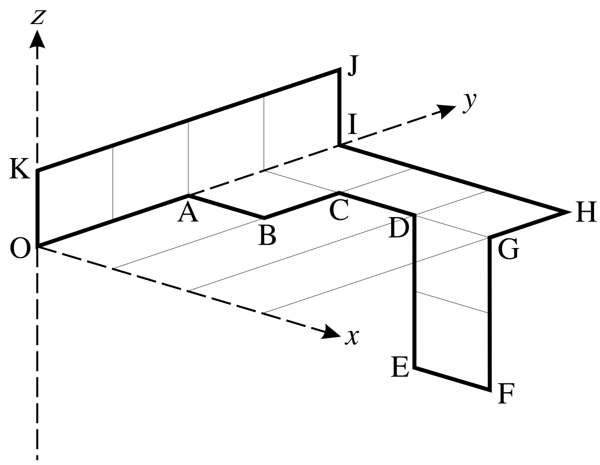

Fig. 3.2

(iii) Show that, referred to the axes given in Fig. 3.2, the centre of mass of the bracket has coordinates (1, 2.7, 0). **[5]**

The bracket is now hung freely in equilibrium from a string attached to O.

(iv) Calculate the angle between the edge OI and the vertical. **[4]**

4 **(a)** A parachutist and her equipment have a combined mass of 80 kg. During a descent where the parachutist loses 1600 m in height, her speed reduces from $V \text{ m s}^{-1}$ to 6 m s^{-1} and she does 1.3×10^6 J of work against resistances.

Use an energy method to calculate the value of V. **[5]**

(b) A vehicle of mass 800 kg is climbing a hill inclined at θ to the horizontal, where $\sin \theta = 0.1$. At one time the vehicle has a speed of 8 m s^{-1} and is accelerating up the hill at 0.25 m s^{-2} against a resistance of 1150 N.

(i) Show that the driving force on the vehicle is 2134 N and calculate its power at this time. **[7]**

The vehicle is pulling a sledge, of mass 300 kg, which is sliding up the hill. The sledge is attached to the vehicle by a light, rigid coupling parallel to the slope. The force in the coupling is 900 N.

(ii) Assuming that the only resistance to the motion of the sledge is due to friction, calculate the coefficient of friction between the sledge and the ground. **[6]**

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Friday 13 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4762 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 A bus of mass 8 tonnes is driven up a hill on a straight road. On one part of the hill, the power of the driving force on the bus is constant at 20 kW for one minute.

(i) Calculate how much work is done by the driving force in this time. **[2]**

During this minute the speed of the bus increases from $8\,\text{m s}^{-1}$ to $12\,\text{m s}^{-1}$ and, in addition to the work done against gravity, 125 000 J of work is done against the resistance to motion of the bus parallel to the slope.

(ii) Calculate the change in the kinetic energy of the bus. **[3]**

(iii) Calculate the vertical displacement of the bus. **[4]**

On another stretch of the road, a driving force of power 26 kW is required to propel the bus up a slope of angle θ to the horizontal at a constant speed of $6.5\,\text{m s}^{-1}$, against a resistance to motion of 225 N parallel to the slope.

(iv) Calculate the angle θ. **[6]**

The bus later travels up the same slope of angle θ to the horizontal at the same constant speed of $6.5\,\text{m s}^{-1}$ but now against a resistance to motion of 155 N parallel to the slope.

(v) Calculate the power of the driving force on the bus. **[2]**

2 The shaded region shown in Fig. 2.1 is cut from a sheet of thin rigid uniform metal; LBCK and EFHI are rectangles; EF is perpendicular to CK. The dimensions shown in the figure are in centimetres. The O*y* and O*z* axes are also shown.

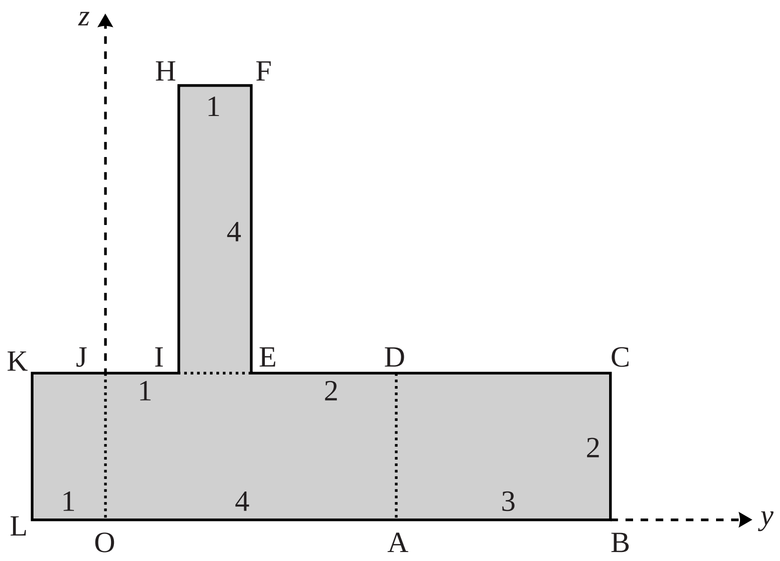

Fig. 2.1

(i) Calculate the coordinates of the centre of mass of the metal shape referred to the axes shown in Fig. 2.1. **[4]**

The metal shape is freely suspended from the point H and hangs in equilibrium.

(ii) Calculate the angle that HI makes with the vertical. **[4]**

The metal shape is now folded along OJ, AD and EI to give the object shown in Fig. 2.2; LOJK, ABCD and IEFH are all perpendicular to OADJ; LOJK and ABCD are on one side of OADJ and IEFH is on the other side of it.

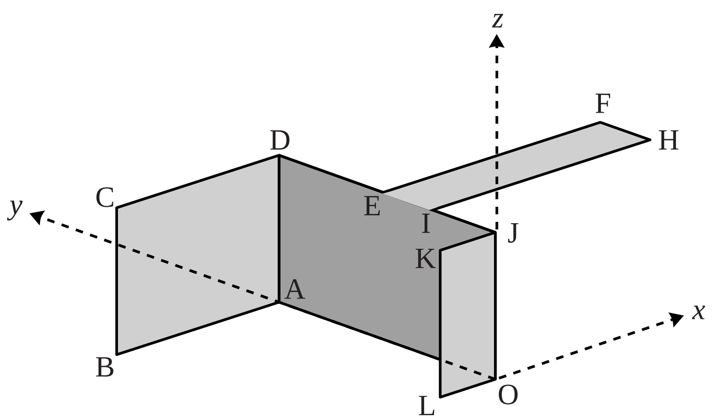

Fig. 2.2

(iii) Referred to the axes shown in Fig. 2.2, show that the *x*-coordinate of the centre of mass of the object is –0.1 and find the other two coordinates of the centre of mass. **[6]**

The object is placed on a rough inclined plane with LOAB in contact with the plane. OL is parallel to a line of greatest slope of the plane with L higher than O. The object does not slip but is on the point of tipping about the edge OA.

(iv) Calculate the angle of OL to the horizontal. **[4]**

3 A thin rigid non-uniform beam AB of length 6 m has weight 800 N. Its centre of mass, G, is 2 m from B.

Initially the beam is horizontal and in equilibrium when supported by a small round peg at C, 1 m from A, and a light vertical wire at B. This situation is shown in Fig. 3.1 where the lengths are in metres.

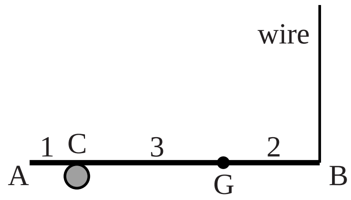

Fig. 3.1

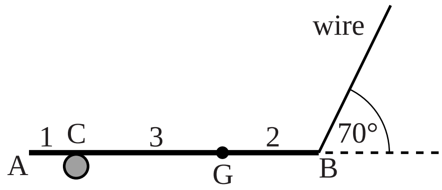

Fig. 3.2

(i) Calculate the tension in the wire and the normal reaction of the peg on the beam. **[4]**

The beam is now held horizontal and in equilibrium with the wire at 70° to the horizontal, as shown in Fig. 3.2. The peg at C is rough and still supports the beam 1 m from A. The beam is on the point of slipping.

(ii) Calculate the new tension in the wire.

Calculate also the coefficient of friction between the peg and the beam. **[7]**

The beam is now held in equilibrium at 30° to the vertical with the wire at $\theta°$ to the beam, as shown in Fig. 3.3. A new small **smooth** peg now makes contact with the beam at C, still 1 m from A. The tension in the wire is now T N.

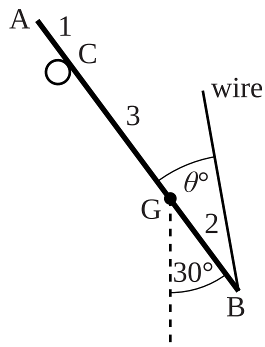

Fig. 3.3

(iii) By taking moments about C, resolving in a suitable direction and obtaining two equations in terms of θ and T, or otherwise, calculate θ and T. **[7]**

4 **(a)** A large nail of mass 0.02 kg has been driven a short distance horizontally into a fixed block of wood, as shown in Fig. 4.1, and is to be driven horizontally further into the block. The wood produces a constant resistance of 2.43 N to the motion of the nail. The situation is modelled by assuming that linear momentum is conserved when the nail is struck, that all the impacts with the nail are direct and that the head of the nail never reaches the wood.

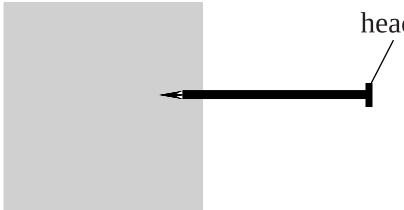

Fig. 4.1

The nail is first struck by an object of mass 0.1 kg that is moving parallel to the nail with linear momentum of magnitude 0.108 N s. The object becomes firmly attached to the nail.

(i) Calculate the speed of the nail and object immediately after the impact. **[2]**

(ii) Calculate the time for which the nail and object move, and the distance they travel in that time. **[4]**

On a second attempt to drive in the nail, it is struck by the same object of mass 0.1 kg moving parallel to the nail with the same linear momentum of magnitude 0.108 N s. This time the object does not become attached to the nail and after the contact is still moving parallel to the nail. The coefficient of restitution in the impact is $\frac{1}{3}$.

(iii) Calculate the speed of the nail immediately after this impact. **[6]**

(b) A small ball slides on a smooth horizontal plane and bounces off a smooth straight vertical wall. The speed of the ball is u before the impact and, as shown in Fig. 4.2, the impact turns the path of the ball through 90°. The coefficient of restitution in the collision between the ball and the wall is e. Before the collision, the path is inclined at α to the wall.

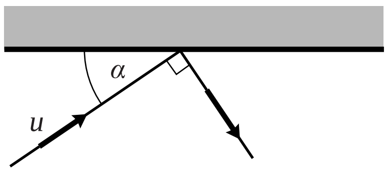

Fig. 4.2

(i) Write down, in terms of u, e and α, the components of the velocity of the ball parallel and perpendicular to the wall before and after the impact. **[3]**

(ii) Show that $\tan \alpha = \dfrac{1}{\sqrt{e}}$. **[3]**

(iii) Hence show that $\alpha \geqslant 45°$. **[1]**

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Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4762 Mechanics 2

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1 **(a)** A stone of mass 0.6 kg falls vertically 1.5 m from A to B against resistance. Its downward speeds at A and B are $5.5\,\text{m}\,\text{s}^{-1}$ and $7.5\,\text{m}\,\text{s}^{-1}$ respectively.

(i) Calculate the change in kinetic energy and the change in gravitational potential energy of the stone as it falls from A to B. **[3]**

(ii) Calculate the work done against resistance to the motion of the stone as it falls from A to B. **[2]**

(iii) Assuming the resistive force is constant, calculate the power with which the resistive force is retarding the stone when it is at A. **[4]**

(b) A uniform plank is inclined at 40° to the horizontal. A box of mass 0.8 kg is on the point of sliding down it. The coefficient of friction between the box and the plank is μ.

(i) Show that $\mu = \tan 40°$. **[4]**

The plank is now inclined at 20° to the horizontal.

(ii) Calculate the work done when the box is pushed 3 m up the plank, starting and finishing at rest. **[5]**

2 The rigid object shown in Fig. 2.1 is made of thin non-uniform rods. ABC is a straight line; BC, BE and ED form three sides of a rectangle. The centre of mass of the object is at G. The lengths are in centimetres. The weight of the object is 15 N.

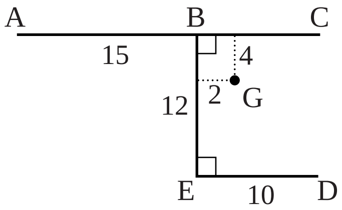

Fig. 2.1

Initially, the object is suspended by light vertical strings attached to B and to C and hangs in equilibrium with AC horizontal.

(i) Calculate the tensions in each of the strings. **[4]**

In a new situation the strings are removed. The object can rotate freely in a vertical plane about a fixed horizontal axis through A and perpendicular to ABCDE. The object is held in equilibrium with AC horizontal by a force of magnitude T N in the plane ABCDE acting at C at an angle of 30° to CA. This situation is shown in Fig. 2.2.

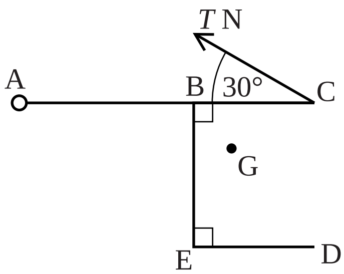

Fig. 2.2

(ii) Calculate T.

Calculate also the magnitude of the force exerted on the object by the axis at A. **[6]**

The object is now placed on a rough horizontal table and is in equilibrium with ABCDE in a vertical plane and DE in contact with the table. The coefficient of friction between the edge DE and the table is 0.65. A force of slowly increasing magnitude (starting at 0 N) is applied at A in the direction AB. Assume that the object remains in a vertical plane.

(iii) Determine whether the object slips before it tips. **[6]**

3 **(a)** You are given that the position of the centre of mass, G, of a right-angled triangle cut from thin uniform material in the position shown in Fig. 3.1 is at the point $(\frac{1}{3}a, \frac{1}{3}b)$.

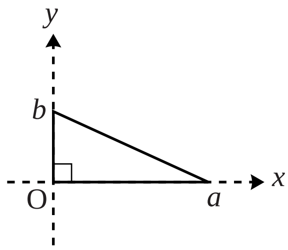

Fig. 3.1

A plane thin uniform sheet of metal is in the shape OABCDEFHIJO shown in Fig. 3.2. BDEA and CDIJ are rectangles and FEH is a right angle. The lengths of the sides are shown with each unit representing 1 cm.

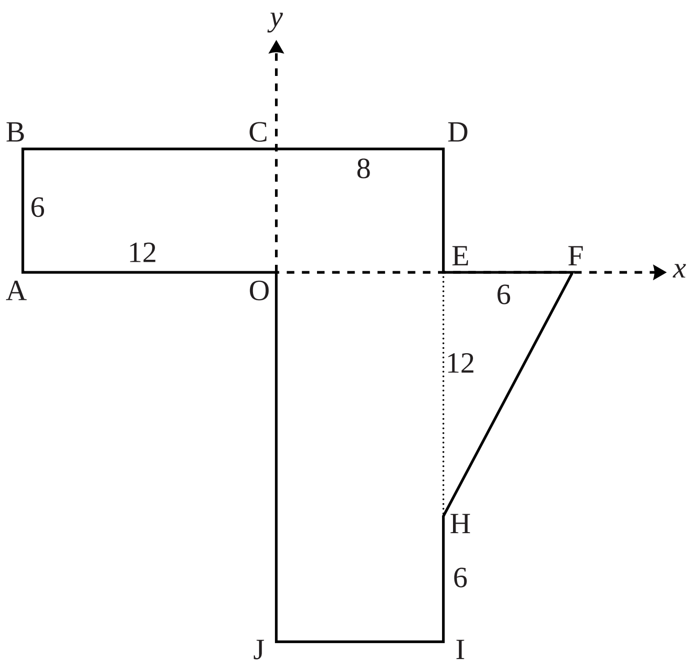

Fig. 3.2

(i) Calculate the coordinates of the centre of mass of the metal sheet, referred to the axes shown in Fig. 3.2. **[5]**

The metal sheet is freely suspended from corner B and hangs in equilibrium.

(ii) Calculate the angle between BD and the vertical. **[4]**

(b) Part of a framework of light rigid rods freely pin-jointed at their ends is shown in Fig. 3.3. The framework is in equilibrium.

All the rods meeting at the pin-joints at A, B and C are shown. The rods connected to A, B and C are connected to the rest of the framework at P, Q, R, S and T.

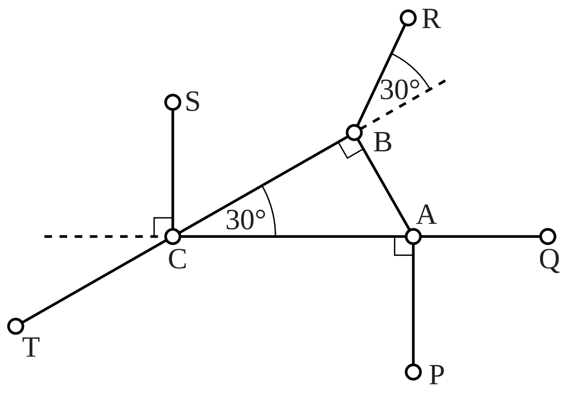

Fig. 3.3

There is a tension of 18 N in rod AP and a thrust (compression) of 5 N in rod AQ.

(i) Show the forces internal to the rods acting on the pin-joints at A, B and C. **[2]**

(ii) Calculate the forces internal to the rods AB, BC and CA, stating whether each rod is in tension or compression. [You may leave your answers in surd form. Your working in this part should be consistent with your diagram in part **(i)**.] **[7]**

4 P and Q are circular discs of mass 3 kg and 10 kg respectively which slide on a smooth horizontal surface. The discs have the same diameter and move in the line joining their centres with no resistive forces acting on them. The surface has vertical walls which are perpendicular to the line of centres of the discs. This information is shown in Fig. 4 together with the direction you should take as being positive.

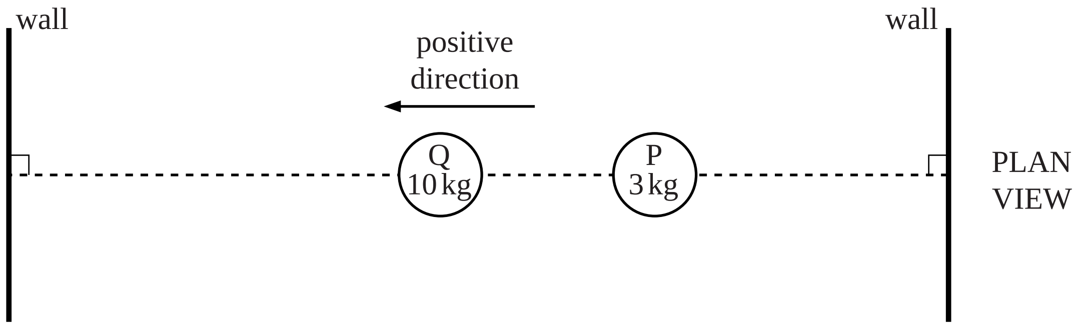

Fig. 4

(i) For what time must a force of 26 N act on P to accelerate it from rest to $13\,\mathrm{m\,s^{-1}}$? **[2]**

P is travelling at $13\,\mathrm{m\,s^{-1}}$ when it collides with Q, which is at rest. The coefficient of restitution in this collision is e.

(ii) Show that, after the collision, the velocity of P is $(3 - 10e)\,\mathrm{m\,s^{-1}}$ and find an expression in terms of e for the velocity of Q. **[7]**

(iii) For what set of values of e does the collision cause P to reverse its direction of motion? **[2]**

(iv) Determine the set of values of e for which P has a greater speed than Q immediately after the collision. **[4]**

You are now given that $e = \frac{1}{2}$. After P and Q collide with one another, each has a perfectly elastic collision with a wall. P and Q then collide with one another again and in this second collision they stick together (coalesce).

(v) Determine the common velocity of P and Q. **[4]**

(vi) Determine the impulse of Q on P in this collision. **[1]**

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Monday 14 January 2013 – Morning

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4762/01 Mechanics 2

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1 **(a)** Fig. 1.1 shows the velocities of a tanker of mass 120 000 tonnes before and after it changed speed and direction.

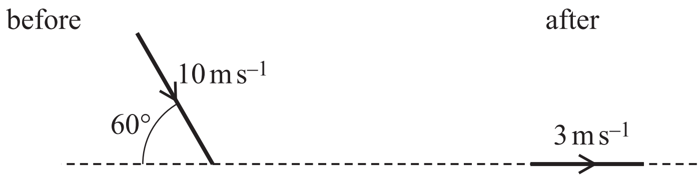

Fig. 1.1

Calculate the magnitude of the impulse that acted on the tanker. **[4]**

(b) An object of negligible size is at rest on a horizontal surface. It explodes into two parts, P and Q, which then slide along the surface.

Part P has mass 0.4 kg and speed $6\,\mathrm{m\,s^{-1}}$. Part Q has mass 0.5 kg.

(i) Calculate the speed of Q immediately after the explosion. State how the directions of motion of P and Q are related. **[2]**

The explosion takes place at a distance of 0.75 m from a raised vertical edge, as shown in Fig. 1.2. P travels along a line perpendicular to this edge.

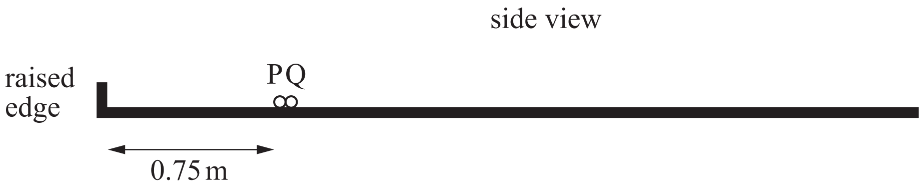

Fig. 1.2

After the explosion, P has a perfectly elastic direct collision with the raised edge and then collides again directly with Q. The collision between P and Q occurs $\frac{2}{3}$ s after the explosion. Both collisions are instantaneous.

The contact between P and the surface is smooth but there is a constant frictional force between Q and the surface.

(ii) Show that Q has speed $2.7\,\mathrm{m\,s^{-1}}$ just before P collides with it. **[4]**

(iii) Calculate the coefficient of friction between Q and the surface. **[4]**

(iv) Given that the coefficient of restitution between P and Q is $\frac{1}{8}$, calculate the speed of Q immediately after its collision with P. **[5]**

2 This question is about 'kart gravity racing' in which, after an initial push, unpowered home-made karts race down a sloping track.

The moving karts have only the following resistive forces and these both act in the direction opposite to the motion.

- A force R, called rolling friction, with magnitude $0.01Mg\cos\theta$ N where M kg is the mass of the kart and driver and θ is the angle of the track with the horizontal
- A force F of varying magnitude, due to air resistance

A kart with its driver has a mass of 80 kg.

One stretch of track slopes uniformly downwards at 4° to the horizontal. The kart travels 12 m down this stretch of track. The total work done by the kart against both rolling friction and air resistance is 455 J.

(i) Calculate the work done against air resistance. **[4]**

(ii) During this motion, the kart's speed increases from $2\,\text{m s}^{-1}$ to $v\,\text{m s}^{-1}$. Use an energy method to calculate v. **[5]**

To reach the starting line, the kart (with the driver seated) is pushed *up* a slope against rolling friction and air resistance.

At one point the slope is at 5° to the horizontal, the air resistance is 15 N, the acceleration of the kart is $1.5\,\text{m s}^{-2}$ up the slope and the power of the pushing force is 405 W.

(iii) Calculate the speed of the kart at this point. **[7]**

3 The object shown shaded in Fig. 3.1 is cut from a flat sheet of thin rigid uniform material; LMJK, OAIJ, AEFH and CDEB are rectangles. The grid-lines in Fig. 3.1 are 1 cm apart.

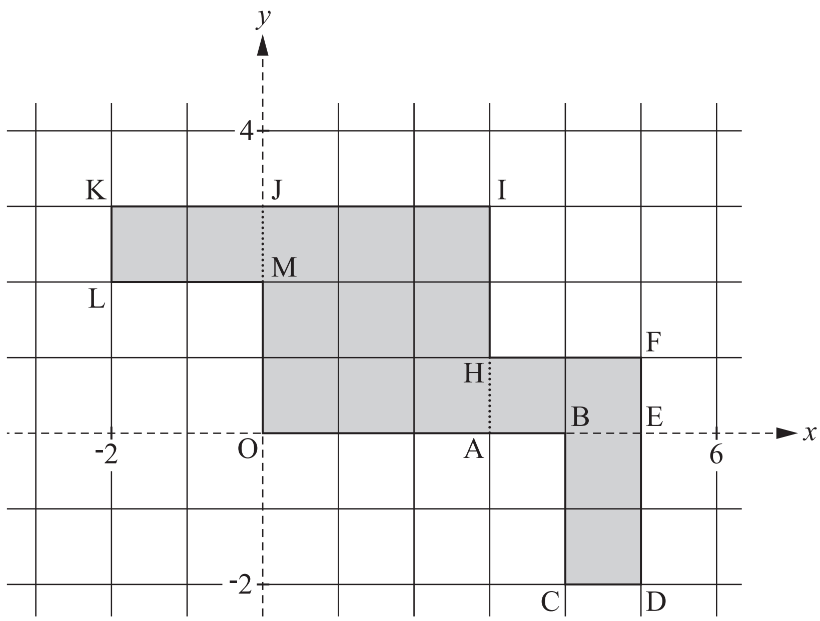

Fig. 3.1

(i) Calculate the coordinates of the centre of mass of the object referred to the axes shown in Fig. 3.1. **[5]**

The object is freely suspended from the point K and hangs in equilibrium.

(ii) Calculate the angle that KI makes with the vertical. **[4]**

The mass of the object is 0.3 kg.

A particle of mass m kg is attached to the object at a point on the line OJ so that the new centre of mass is at the centre of the square OAIJ.

(iii) Calculate the value of m and the position of the particle referred to the axes shown in Fig. 3.1. **[6]**

The extra particle is now removed and the object shown in Fig. 3.1 is folded: LMJK is folded along JM so that it is perpendicular to OAIJ; ABCDEFH is folded along AH so that it is perpendicular to OAIJ and on the same side of OAIJ as LMJK. The folded object is placed on a horizontal table with the edges KL and FED in contact with the table. A plan view and a 3D representation are shown in Fig. 3.2.

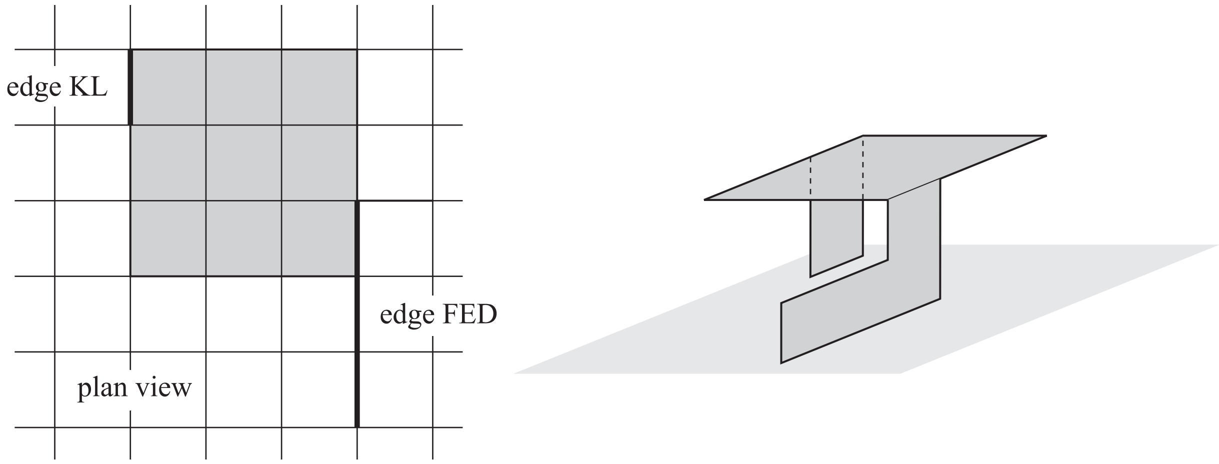

Fig. 3.2

(iv) On the plan, indicate the region corresponding to positions of the centre of mass for which the folded object is stable.

You are given that the x-coordinate of the centre of mass of the folded object is 1.7. Determine whether the object is stable. **[4]**

4 A rigid thin uniform rod AB with length 2.4 m and weight 30 N is used in different situations.

(i) In the first situation, the rod rests on a small support 0.6 m from B and is held horizontally in equilibrium by a vertical string attached to A, as shown in Fig. 4.1.

Fig. 4.1

Calculate the tension in the string and the force of the support on the rod. **[4]**

(ii) In the second situation, the rod rests in equilibrium on the point of slipping with end A on a horizontal floor and the rod resting at P on a fixed block of height 0.9 m, as shown in Fig. 4.2. The rod is perpendicular to the edge of the block on which it rests and is inclined at θ to the horizontal.

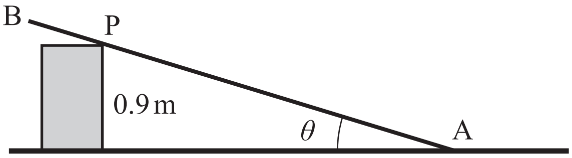

Fig. 4.2

(*A*) Suppose that the contact between the block and the rod is rough with coefficient of friction 0.6 and contact between the end A and the floor is smooth.

Show that $\tan\theta = 0.6$. **[5]**

(*B*) Suppose instead that the contact between the block and the rod is smooth and the contact between the end A and the floor is rough. The rod is now in limiting equilibrium at a different angle θ such that the distance AP is 1.5 m.

Calculate the normal reaction of the block on the rod.

Calculate the coefficient of friction between the rod and the floor. **[9]**

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4762/01 Mechanics 2

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- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** In this part-question, all the objects move along the same straight line on a smooth horizontal plane. All their collisions are direct.

The masses of the objects P, Q and R and the initial velocities of P and Q (but not R) are shown in Fig. 1.1.

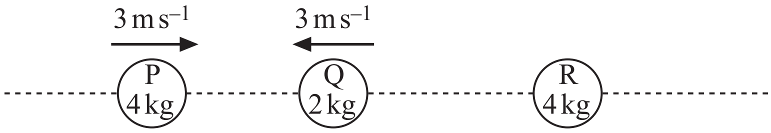

Fig. 1.1

A force of 21 N acts on P for 2 seconds in the direction PQ. P does not reach Q in this time.

(i) Calculate the speed of P after the 2 seconds. **[2]**

The force of 21 N is removed after the 2 seconds. When P collides with Q they stick together (coalesce) to form an object S of mass 6 kg.

(ii) Show that immediately after the collision S has a velocity of $8\,\text{m s}^{-1}$ towards R. **[2]**

The collision between S and R is elastic with coefficient of restitution $\frac{1}{4}$. After the collision, S has a velocity of $5\,\text{m s}^{-1}$ in the direction of its motion before the collision.

(iii) Find the velocities of R before and after the collision. **[6]**

(b) **In this part-question take $g = 10$.**

A particle of mass 0.2 kg is projected vertically downwards with initial speed $5\,\text{m s}^{-1}$ and it travels 10 m before colliding with a fixed smooth plane. The plane is inclined at α to the vertical where $\tan\alpha = \frac{3}{4}$. Immediately after its collision with the plane, the particle has a speed of $13\,\text{m s}^{-1}$. This information is shown in Fig. 1.2. Air resistance is negligible.

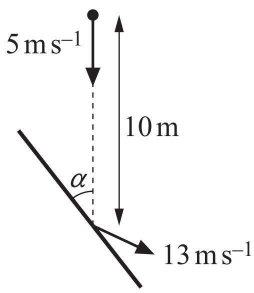

Fig. 1.2

(i) Calculate the angle between the direction of motion of the particle and the plane immediately after the collision.

Calculate also the coefficient of restitution in the collision. **[8]**

(ii) Calculate the magnitude of the impulse of the plane on the particle. **[2]**

2 A fairground ride consists of raising vertically a bench with people sitting on it, allowing the bench to drop and then bringing it to rest using brakes. Fig. 2 shows the bench and its supporting tower. The tower provides lifting and braking mechanisms. The resistances to motion are modelled as having a constant value of 400 N whenever the bench is moving up or down; the only other resistance to motion comes from the action of the brakes.

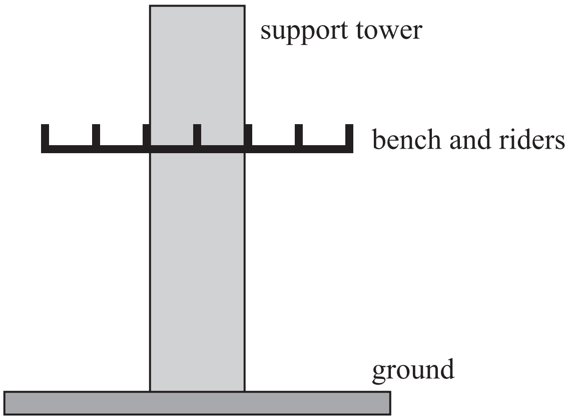

Fig. 2

On one occasion, the mass of the bench (with its riders) is 800 kg.

With the brakes not applied, the bench is lifted a distance of 6 m in 12 seconds. It starts from rest and ends at rest.

(i) Show that the work done in lifting the bench in this way is 49 440 J and calculate the average power required. **[4]**

For a short period while the bench is being lifted it has a constant speed of $0.55\,\text{m}\,\text{s}^{-1}$.

(ii) Calculate the power required during this period. **[3]**

With neither the lifting mechanism nor the brakes applied, the bench is now released from rest and drops 3 m.

(iii) Using an energy method, calculate the speed of the bench when it has dropped 3 m. **[4]**

The brakes are now applied and they halve the speed of the bench while it falls a further 0.8 m.

(iv) Using an energy method, calculate the work done by the brakes. **[5]**

3 Fig. 3.1 shows a rigid, thin, **non-uniform** 20 cm by 80 cm rectangular panel ABCD of weight 60 N that is in a vertical plane. Its dimensions and the position of its centre of mass, G, are shown in centimetres. The panel is free to rotate about a fixed horizontal axis through A perpendicular to its plane; the panel rests on a small smooth fixed peg at B positioned so that AB is at 40° to the horizontal. A horizontal force in the plane of ABCD of magnitude P N acts at D away from the panel.

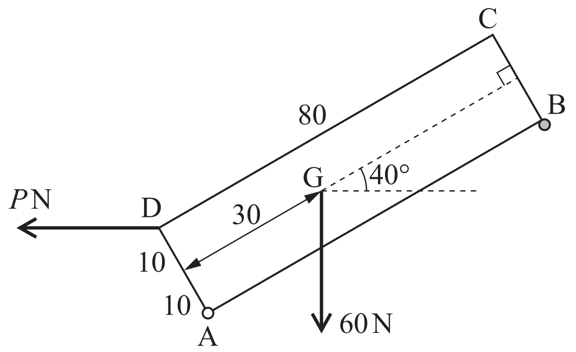

Fig. 3.1

(i) Show that the clockwise moment of the weight about A is 9.93 N m, correct to 3 significant figures. **[3]**

(ii) Calculate the value of P for which the panel is on the point of turning about the axis through A. **[2]**

(iii) In the situation where $P = 0$, calculate the vertical component of the force exerted on the panel by the axis through A. **[4]**

The panel is now placed on a line of greatest slope of a rough plane inclined at 40° to the horizontal. The panel is at all times in a vertical plane. A horizontal force in the plane ABCD of magnitude 200 N acts at D towards the panel. This situation is shown in Fig. 3.2.

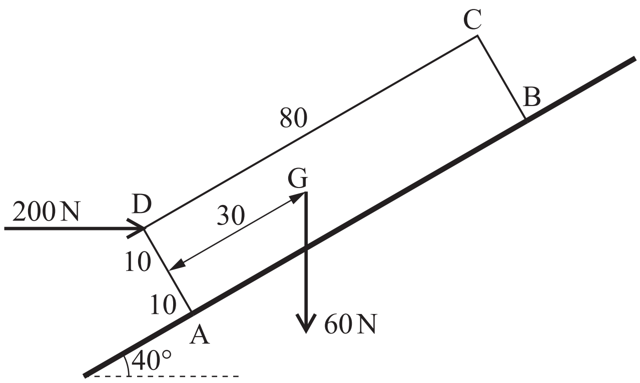

Fig. 3.2

(iv) Given that the panel is moving up the plane with acceleration up the plane of 1.75 m s^{-2}, calculate the coefficient of friction between the panel and the plane. **[8]**

4 **(a)** Fig. 4.1 shows a framework constructed from 4 uniform heavy rigid rods OP, OQ, PR and RS, rigidly joined at O, P, Q, R and S and with OQ perpendicular to PR. Fig. 4.1 also shows the dimensions of the rods and axes Ox and Oy: the units are metres.

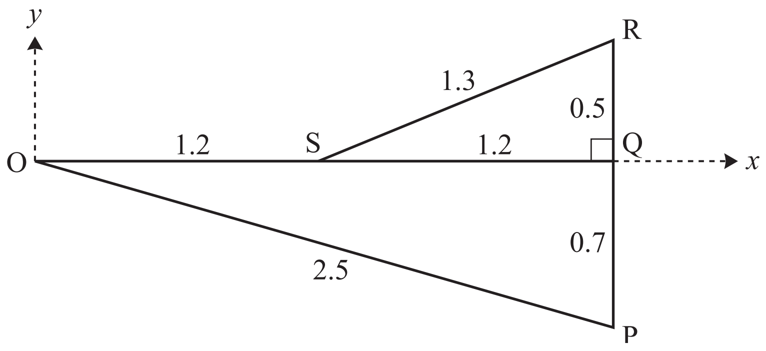

Fig. 4.1

Each rod has a mass of 0.8 kg per metre.

(i) Show that, referred to the axes in Fig. 4.1, the x-coordinate of the centre of mass of the framework is 1.5 and calculate the y-coordinate. **[5]**

The framework is freely suspended from S and a small object of mass m kg is attached to it at O. The framework is in equilibrium with OQ horizontal.

(ii) Calculate m. **[3]**

[Question 4 is continued overleaf.]

(b) Fig. 4.2 shows a framework in equilibrium in a vertical plane. The framework is made from 5 light, rigid rods OP, OQ, OR, PQ and QR. Its dimensions are indicated. PQ is horizontal and OR vertical.

The rods are freely pin-jointed to each other at O, P, Q and R. The pin-joint at O is fixed to a wall.

Fig. 4.2 also shows the external forces acting on the framework: there are vertical loads of 120 N and 60 N at Q and P respectively; a horizontal string attached to Q has tension T N; horizontal and vertical forces X N and Y N act on the framework from the pin-joint at O.

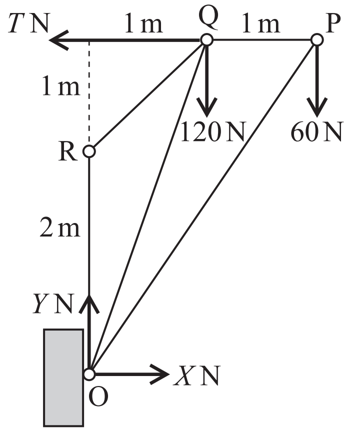

Fig. 4.2

(i) By considering only the pin-joint at R, explain why the rods OR and RQ must have zero internal force. **[2]**

(ii) Find the values of T, X and Y. **[3]**

(iii) Using the diagram in your printed answer book, show all the forces acting on the pin-joints, including those internal to the rods. **[1]**

(iv) Calculate the forces internal to the rods OP and PQ, stating whether each rod is in tension or compression (thrust). [You may leave answers in surd form. Your working in this part should correspond to your diagram in part **(iii)**.] **[5]**

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